An Economic Anatomy of Optimal Climate Policy

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An Economic Anatomy of Optimal Climate Policy

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This paper introduces geoengineering into an optimal control model of climate change economics. Together with mitigation and adaptation, carbon and solar geoengineering span the universe of possible climate policies. We show in the context of our model that: (i) a carbon tax is the optimal response to the unpriced carbon externality only if it equals the marginal cost of carbon geoengineering; (ii) the introduction of solar geoengineering leads to higher emissions yet lower temperatures, and, thus, increased welfare; and (iii) solar geoengineering, in effect, is a public goods version of adaptation that also lowers temperatures.

JEL: D90, O44, Q48, Q54, Q58

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Conventional economic wisdom says that the optimal climate policy is to follow the logic of Pigou (1920) and price carbon dioxide (CO_2) and other greenhouse-gas emissions[1] at their marginal costs to society: internalize the negative externality, and get out of the way.[2] While Pigou is right, the conventional wisdom is wrong,

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[1] While there are important differences between long-lived climate forcers, like CO_2, and short-lived climate forcers like methane (Shindell et al., 2017), we here focus on CO_2, and henceforth use "CO_2" as a shortcut for greenhouse-gas emissions. Any mention of, e.g., "carbon stock" for expositional expediency should, thus, be interpreted as "CO_2 stock."

[2] Some invoke Coase (1960) instead of Pigou (1920). As property rights to the Earth's atmosphere

or at least it is limiting. For one, it is limiting because of the unpriced, positive learning-by-doing externality inherent in the adoption of new, cleaner technologies (e.g., Acemoglu et al., 2012).[3] A second, even more fundamental reason for why the conventional wisdom is wrong, is that the global warming effects of CO_2 emissions do not enter the welfare function directly. The effects instead propagate through a long causal chain, with emissions affecting concentrations, concentrations affecting temperatures, and temperatures affecting damages affecting human welfare. Each link engenders its own possible interventions. The implications of these interventions, and their interactions, is the focus of our paper.

We show here that any optimal climate policy portfolio includes four distinct interventions. Society can avoid emitting CO_2 in the first place: *mitigation.* It can adapt to new climate realities: *adaptation.* It can extract carbon from the air: *carbon geoengineering.*[4] Lastly, it can attempt to affect temperatures directly: *solar geoengineering.*[5]. The bulk of the climate economics literature focused on mitigation (e.g., Acemoglu et al., 2012; Goulder and Pizer, 2006; Nordhaus, 2013; Stern, 2007), with some entries on adaptation (e.g., Bruin, Dellink and Tol, 2009; Kahn, 2013; Mendelsohn, 2012). Carbon geoengineering occupies a niche at once mundane and unique: economic models often fail to call it out because it merely looks like "expensive mitigation." It is not. In fact, it is the only possible intervention that attacks the root cause of climate change—too much CO_2 in the atmosphere—without decreasing emissions, and the only intervention that allows for actually decreasing the stock of atmospheric CO_2 without simply waiting for slow, natural processes to do so. The small economic literature on solar geoengi-

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are poorly defined, to say the least, the solution all but requires a Pigouvian tax instead of Coasian bargaining. We would argue that Coase himself agreed with that assessment (Glaeser, Johnson and Shleifer, 2001).

[3] The existence of a second, positive externality and the policy interplay with CO_2 pricing leads to important political economy considerations (e.g., Acemoglu et al., 2016; Bennear and Stavins, 2007; Meckling et al., 2015; Wagner et al., 2015).

[4] Carbon geoengineering is commonly also referred to as "air capture" or "carbon dioxide removal" (CDR) and, confusingly, sometimes as "direct carbon removal" (DCR). See NRC (2015*a*) for a survey of methods and their implications.

[5] Solar geoengineering, in turn, comes under various names including "solar radiation management," "albedo modification," "climate remediation," and sometimes simply "geoengineering" or "climate engineering" as a catch-all term (e.g., Keith, 2000; NRC, 2015*b*).

neering, in turn, often focuses on it in isolation, with a few exceptions considering both solar geoengineering and mitigation as part of a mixed portfolio (Moreno-Cruz, 2015; Moreno-Cruz and Keith, 2012; Heutel, Moreno-Cruz and Shayegh, 2016). Our model attempts to capture the pertinent characteristics of each of these possible policy interventions in their most stylized form.

Mitigation is slow and costly.[6] This makes it the poster child of the free-rider problem, as countries and individuals seek to postpone costly emissions reduction measures with the intention of inducing higher mitigation efforts by others (e.g., Nordhaus, 2015; Weitzman, 2016; Cramton et al., 2017). While our model employs one representative agent and, thus, does not explicitly capture the free-rider problem *per se*, it very much captures the implications. We assume that the only way to create appropriate incentives for mitigation is via a broad-based CO_2 tax. In practice, that "tax," of course, is often anything but, resembling a shadow price on emissions.[7] Mitigation alone, however, is not enough for an optimal solution, largely due to inertia in the climate system. Global average temperatures have already risen by around 1^oC since before the industrial revolution, with almost as much additional warming baked in due to elevated atmospheric CO_2 concentrations (IPCC, 2013; Friedlingstein et al., 2006). That points to the all-important time element in climate policy. It also necessitates interventions further along the chain.

Carbon geoengineering mimics mitigation in important ways. It is as slow as and often costlier than mitigation. It perfectly and directly compensates for increased CO_2 concentrations (Heutel, Moreno-Cruz and Shayegh, 2016; NRC, 2015*a*). It differs from mitigation in that it is not limited by the scale of the

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[6] "Costly," of course, is relative. The question relevant for policy is "costly" compared to what? All that said, standard economic models typically assume it is costly. It need not be. Along many metrics, costs are decreasing fast. See footnote 10.

[7] It could either take the form of a quantity-based instrument (Dales, 1968; Weitzman, 1974; Keohane, 2009), an implicit price instituted via other policy instruments (e.g., Bennear and Stavins, 2007), a direct tax (e.g., Metcalf, 2009), or a combination of two or more instruments (e.g., Pizer, 2002; Fankhauser, Hepburn and Park, 2010). More often than not, it comes in the form of deliberate technological interventions. In any case, political economy considerations call for clear thinking around policy sequencing toward anything close to a first-best outcome (Wagner et al., 2015).

economy. Unlike mitigation alone, it can lead to net-negative changes in the atmospheric CO_2 stock in any given year, much faster than natural processes. "Fast," of course, is relative. Inherent inertia in the climate system means that even with emissions reductions and carbon geoengineering at scales leading to net negative emissions by mid-century, temperatures and sea levels would rise for decades and centuries to come (Matthews et al., 2009; Solomon et al., 2009), pointing to the need to further interventions further down the climate system chain.

Solar geoengineering is quicker and cheaper than either mitigation or carbon geoengineering (NRC, 2015*b*; Moreno-Cruz, Ricke and Wagner, 2017). It perfectly and directly compensates for increased temperatures, though temperatures themselves are an imperfect proxy for climate damages. It also intervenes further down the climate system chain, not tackling excess CO_2 in the first place. It is also inexpensive. Preliminary estimates point to direct costs in the order of billions of dollars a year to turn down global average temperatures to preindustrial levels (McClellan, Keith and Apt, 2012), rather than trillions, as is the case for mitigation and carbon geoengineering (NRC, 2015*a*). It can be implemented without full participation (Barrett, 2008, 2014). Instead of sharing classic free-rider properties with mitigation and carbon geoengineering, solar geoengineering exhibits a "free-driver" effect (Moreno-Cruz, 2015; Wagner and Weitzman, 2012, 2015; Weitzman, 2015). This creates the distinct possibility that solar geoengineering is oversupplied in the future. It can also be undersupplied if the country with the means to implement solar geoengineering chooses not to do so (Moreno-Cruz and Smulders, 2017). Thus, while a CO_2 tax is necessary to motivate mitigation and carbon geoengineering, a "temperature tax" is not.[8]

Adaptation, meanwhile, is imperfect and private. In fact, it is doubly imperfect, as it has no direct effect on either CO_2 stocks or on temperatures. While

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[8] The combination, a CO_2 tax pegged to temperatures (McKitrick, 2011), is similarly misguided for the simple reason that inherent inertia in the climate system delays feedback by centuries.

it affects provisions of public goods—from migration to mitigation—adaptation itself is rival and excludable, making it a classic private good (Samuelson, 1954).[9] Depending on the scale of adaptation, it can be relatively quick and cheap—think a second air conditioner—or slow and expensive—think moving entire cities to higher land (Desmet and Rossi-Hansberg, 2015). In any case, adaptation should not be confused with "suffering." Adaptation is deliberate (Kahn, 2013). Suffering, a loss in welfare because of inadequate climate policy interventions, is not.

Put back into the language of the chain from emissions to human welfare, only mitigation propagates throughout the entire chain. The other three interventions are aimed at breaking otherwise believed to be firm links: carbon geoengineering breaks the link between emissions and concentrations; solar geoengineering breaks the link between concentrations and temperatures; adaptation breaks the link between temperature and damages. What then is the best way to combine these four instruments to optimally manage climate change?

To address this question, we develop a parsimonious model of climate change economics that captures the main trade-offs associated with all four instruments. Economic output, of which emissions are an important component[10], propagates through the emissions-concentrations-temperatures chain through to damages, which, in turn, lead to reductions in economic output. Mitigation reduces emissions. Mitigation and carbon geoengineering both reduce concentrations. Mitigation, carbon geoengineering, and solar geoengineering reduce temperatures. Mitigation, carbon geoengineering, solar geoengineering, and adaptation reduce the resulting damages.

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[9] Our model with one representative agent does not, in fact, lend itself to a proper analysis of this private goods aspect of adaptation. Doing so necessitates extending the framework to more than one agent.

[10] Breaking the link between economic output and emissions is itself an important goal of climate policy aimed at mitigating emissions in the first place. McKinsey (2009), for example, finds 1 billion tons of CO_2-equivalent emissions reduction opportunities per year that have positive net present value in the United States alone. Allcott and Greenstone (2012) and Gerarden, Newell and Stavins (2015) assess this "energy efficiency gap" without conclusive evidence as to its existence. Gillingham and Palmer (2014) are more positive. A natural extension of our model is to include two goods—one "dirty," one "clean"—and to model the substitutability among them (e.g., Acemoglu et al., 2012, 2016).

The climate system is complex. Climate models, therefore, are often appropriately complex, too. Climate-economy models, meanwhile, typically reduce both the climate and economic systems to their essential components. Nordhaus (1992, 2013)'s Dynamic Integrated Climate-Economy (DICE) model famously includes fewer than twenty main equations in order to calculate the optimal global CO_2 price path.[11] We focus on a partial-equilibrium setting and reduce the climate system to two dynamic equations: one describes the stock of CO_2 in the atmosphere, S; the other focuses on global average temperatures, T, at any given point, based on changes in that stock.[12] The two are intimately linked via the all-important climate sensitivity parameter (e.g., Matthews et al., 2009), which translates a doubling of atmospheric concentrations of CO_2 into global average temperature outcomes—in equilibrium.

The term "equilibrium" itself merits discussion. Climatic and economic systems adjust—and reach equilibrium—on entirely different timescales. The $1.5-4.5^oC$ "likely" range of parameter values for climate sensitivity that is typically used in economic models (Roe and Baker, 2007; Weitzman, 2009*b*; Wagner and Weitzman, 2015), is, in geological terms, the so-called "fast" equilibrium (IPCC, 2013). The "Earth system" equilibrium considering, for example, albedo effects from melting ice sheets is significantly higher at $\sim 4-6^oC$ (Previdi et al., 2013). Considering carbon-cycle feedbacks and other changes on the scale of centuries and millenia would yield a range higher still at $\sim 6-8^oC$. For our analytic solutions, we do not, in fact, care about the specific climate sensitivity numbers and their implications for optimal climate policy. We *do* care about "fast" versus "slow" equilibria to draw a line between what is—and what is not—in equilibrium (or "quasi-equilibrium") at any point in time along the climate system chain. Most of the temperature response that will happen within a century due to added CO_2 in the

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[11] See Nordhaus and Sztorc (2013). For extensive critiques and long lists of well-known limitations, see, among others: Burke et al. (2016); Convery and Wagner (2015); Fisher and Le (2014); Kopp et al. (2016); Pindyck (2013); Stern (2013); Wagner and Weitzman (2015); Weitzman (2009*b*); NAS (2017).

[12] See, e.g., Nordhaus (1991); Golosov et al. (2014).

atmosphere, in fact, happens within a decade.[13] We can, thus, take advantage of the quasi-equilibrium behavior of climate policy over the time frames that matter for policy.

We define *fast equilibrium* as the state of temperature T at any given point in time. In short, on timescales relevant for policy, T reaches its "fast" equilibrium quickly enough for us to to be able to assume that T is, in fact, always in equilibrium—except for the slow-moving response due to changes in the CO_2 stock S. A *slow equilibrium*, in turn, is the state of the climate system when the carbon cycle, and thus T as a function of stock S, is in equilibrium as well.[14] That equilibrium may not happen for centuries or millenia. The exact time does not matter. What matters is that it is one, two, or even three orders of magnitude slower than reaching the fast equilibrium, which we, by comparison, take as happening instantaneously.

The difference between fast and slow equilibria matters to solving our model. It also matters to our fundamental understanding of optimal climate policy. Instead of an optimal control problem with one knob—S—which is assumed to have a direct link to eventual temperature and climate outcomes, we now have a second knob: T. S and T affect economic welfare in distinct ways. Time plays an important role; so do benefits, costs, and risks. Breaking the direct link between S and T also immediately increases the number of policy goals beyond one. That alone all but guarantees that the "conventional wisdom" around a CO_2 tax needs to be overturned. More than one potential policy target necessitates more than one policy intervention.[15]

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[13] "Maximum warming occurs about one decade after a carbon dioxide emission" (Ricke and Caldeira, 2014). Around half of global average warming due to a rapid increase in atmospheric CO_2 happens within a decade, whereas around a quarter happens after a century (Caldeira and Myhrvold, 2013).

[14] See Held et al. (2010) and Cao et al. (2015) on "fast" versus "slow" responses in the climate system, Proistosescu and Huybers (2017) on fast and slow modes of equilibrium—"fast," on geological timescales—climate sensitivity itself, and Nordhaus (1991) and Lemoine and Rudik (2014) for explicit discussions of time and the effects of inertia in climate-economic models. See also Ricke and Caldeira (2014) and Caldeira and Myhrvold (2013) for detailed modeling results. Caldeira and Myhrvold (2012) explore the implications of using temperature as a metric to evalute climate and energy policies.

[15] Mundell (1968, p. 201), in reference to Tinbergen (1952), likens economic policy systems to "'overdetermined' or 'underdetermined' mathematical systems," unless the number of policy goals matches the number of instruments.

I. General Framework

We focus our model on its most essential components. For example, the representative agent's utility function is quasilinear, given by:

$$U(E(t)) + Q_0(t), \tag{1}$$

where $E(t)$ is emissions of CO_2, and $E(t)$ is the consumption of fossil fuels. This assumption is limiting in one important way: it does not allow us to distinguish between "dirty" and "clean" production and, thus, makes reductions in emissions necessarily costly—an oft-stated assumption in economics, albeit one worthy of further exploration.[16] $Q_0(t)$ is the consumption of all other goods in the economy, taken to be the numeraire. The utility function is strictly concave in $E(t)$.

We consider a partial equilibrium model where global aggregate income, $Y(t)$, is exogenous and equal to $Q_0(t)$ plus the costs of fossil fuel extraction, $pE(t)$, damages from climate change D, and costs of climate intervention C.

Climate damages are denoted by $D(\tilde{T}(t), S(t), G(t))$ and are strictly increasing and weakly convex in each of its elements. Climate damages associated with global average surface temperature, $T(t)$, can be reduced with expenditures on adaptation, $A(t)$. $\tilde{T}$ denotes effective temperature, with:

$$\tilde{T}(t) \equiv T(t) - \chi_A A(t). \tag{2}$$

The indicator variable $\chi_A \in \{0, 1\}$ shows if adaptation is available ($\chi_A = 1$) or not ($\chi_A = 0$). For now, this is the only assumed additional structure of the damage function. Its other elements—CO_2 concentrations, $S(t)$, and solar geoengineering, $G(t)$—have no further structural assumptions.

The costs of managing the climate are given by $C(R(t), G(t), A(t))$, where $R(t)$ is the removal of CO_2 from the atmosphere: carbon geoengineering. Costs are

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[16] See footnote 10.

assumed to be strictly increasing and convex in each element. Mitigation takes the form of reductions in emissions, $E(t)$, relative to a business-as-usual level where climate damages are not considered in the economy. The costs of mitigation are measured in terms of forgone utility.

A. Climate System Dynamics

We capture the climate system and its four-link chain from emissions via concentrations and temperatures to damages in two dynamic equations. The first captures the evolution of the CO_2 stock in the atmosphere, $S(t)$:

$$\dot{S}(t) \equiv \frac{dS(t)}{dt} = E(t) - \chi_R R(t) - \delta_S S(t), \quad S(0) = S_0 > 0. \tag{3}$$

The stock of atmospheric CO_2 increases with past emissions that result from the burning of fossil fuels. It decreases due to natural decay, $\delta_S S$, mainly via uptake by oceans.[17] Carbon geoengineering, $R(t)$, decreases $S(t)$, breaking the otherwise direct link between emissions and concentrations. The indicator variable $\chi_R \in \{0,1\}$ shows if carbon geoengineering is available ($\chi_R = 1$) or not ($\chi_R = 0$).

The second dynamic equation captures the evolution of average global surface temperatures, $T(t)$:

$$\dot{T}(t) \equiv \frac{dT(t)}{dt} = \lambda S(t) - \chi_G G(t) - \delta_T T(t), \quad T(0) = T_0 > 0. \tag{4}$$

We link changes in temperature to atmospheric CO_2 stocks via a linearized climate feedback parameter, λ.[18] Note that λ does not stay constant over time. In fact, it changes with the type of ("slow") climate equilibrium assumed. Tradi-

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[17] We assume δ_S to be a constant. It does, in fact, change with ocean alkalinity and other factors (Egleston, Sabine and Morel, 2010). Those, in turn, are affected, for example, by global average temperatures, ocean acidification due to excess CO_2 burden, and solar geoengineering interventions. Most significantly, δ_S is small, pointing to carbon geoengineering R as the only viable option to decrease S on timescales relevant to policy.

[18] Climate feedback λ, in turn, is closely linked to climate sensitivity, via δ_T: Climate sensitivity links changes in temperature, $\Delta T(t)$, with relative changes in the stock, $\Delta lnS(t)$, based on a doubling of $S(t)$; i.e., climate sensitivity $\equiv ln2 \frac{\Delta T(t)}{\Delta lnS(t)}$. Our linearization follows, e.g., Nordhaus (1991).

tionally, that would mean choosing a climate sensitivity parameter range—either the "likely" equilibrium climate sensitivity range of $1.5-4.5^{o}$C, the higher Earth systems range of $\sim\ 4-6^{o}$C, or the one assuming a full equilibrium in the carbon cycle, closer to $\sim\ 6-8^{o}$C. We are largely agnostic as to which climate sensitivity range λ is based on. Even reaching the geologically "fast" equilibrium would take significantly longer – over a century or two – than what we have called the *fast equilibrium* in which we assume temperature to be at any given point. We can, thus, define our fast ("quasi"-)equilibrium in terms of equation (4).

ASSUMPTION 1: *The climate system is always in a fast equilibrium, setting* $\dot{T}(t)=0$*.*

This does not imply that T is constant vis-à-vis S. In fact, $\lambda > 0$ at all times.[19] It does mean that the behavior of T, on timescales relevant to policy, is such that we can set $\dot{T}(t)=0$ at any given point in time without losing too much realism. At time scales of one or two decades, T behaves *as if* it is in equilibrium, in sharp contrast to the evolution of $\dot{S}(t)$, which takes decades, centuries, or millenia to reach what we have called *slow equilibrium.*[20]

Natural changes in temperature via ocean heat transfer are parameterized by δ_T.[21] Moreover, solar geoengineering, $G(t)$, reduces $T(t)$ directly, thus breaking the tight—albeit slow—link from S to T. The indicator variable $\chi_G \in \{0,1\}$ shows if solar geoengineering is available ($\chi_G = 1$) or not ($\chi_G = 0$).

Now that we have introduced all three climate policies beyond mitigation, all of which have their own indicator variable, this is an opportune time to introduce a notational convention that will come in handy when we explore the implications of each. We define χ as an array, indicating whether carbon geoengineering, R, solar geoengineering, G, or adaptation, A—in the order in which they enter the

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[19] The exact value of λ changes with which type of (*slow*) equilibrium we assume we are in—reaching it in decades, centuries, or millenia. See our climate sensitivity discussion in the introductory section.

[20] See footnote 13 and the text around it.

[21] See, e.g., Nævdal and Oppenheimer (2007).

climate system chain—are available:

$$\chi \equiv \begin{pmatrix} \chi_R & \chi_G & \chi_A \end{pmatrix}$$

We will assume that all three technologies beyond mitigation are available for the optimal solution. In the (fast equilibrium) steady-state analysis, distinguishing between the three will be important.

Equations (3) and (4) alone point to many possible extensions of our model, from more complex carbon-cycle dynamics introduced in some climate-economic models (e.g., Golosov et al., 2014), to a full treatment of inertia (e.g., Nordhaus, 1991; Lemoine and Rudik, 2014), to an explicit treatment of uncertainty (e.g., Moreno-Cruz and Keith, 2012; Heutel, Moreno-Cruz and Shayegh, 2016). Note also that here the full effects of solar geoengineering are captured by both $T(t)$ and $G(t)$. $T(t)$ captures solar geoengineering's direct temperature impacts. That representation alone would diminish both its potentially positive effects on other dimensions, and ignore other potentially negative impacts. This misses solar geoengineering's potentially large direct carbon impact (Keith, Wagner and Zabel, 2017). $G(t)$ captures any further positive or negative effects not captured by temperature alone. See, e.g., Moreno-Cruz and Smulders (2017) for a full exploration of solar geoengineering's impacts. All are potentially important extensions of our work. Here we focus on this simple two-equation climate system and their most salient interactions to derive stylized facts and their implications.

B. Optimization Problem

The social planner maximizes the present discounted value of social welfare:

$$\max_{\{E,R,G,A\}} \int_0^\infty \left\{ U(E(t)) + Q(t) \right\} e^{-\rho t} dt, \tag{5}$$

subject to the budget constraint:

$$Y(t) = pE(t) + Q(t) + D(\tilde{T}(t), S(t), G(t)) + C(R(t), G(t), A(t)), \tag{6}$$

and the dynamic climate system equations (3) and (4). This four-equation system covers the full optimization problem. A full analytic solution requires more structural assumptions (see Section III). For now, we explore the optimal solution in the most general terms.

II. Optimal Solution

A. Equilibria, Fast...

To solve our model—the objective function in equation (5), subject to the budget constraint, (6), and the evolution of atmospheric $CO2$ stocks S in (3) and temperature T in (4)—we assume that temperature converges fast to a steady state and is always in the *fast equilibrium*, while the carbon cycle exhibits high inertia and only approaches its steady state in what we have called *slow equilibrium.* T, relative to the slow-moving carbon cycle represented by S, is given by:

$$T(S,G) = \frac{\lambda S - \chi_G G}{\delta_T}. \tag{7}$$

Temperatures unequivocally increase with rising CO_2 and decrease with the use of solar geoengineering, shown, respectively, by $T_S(S,G) = \frac{\lambda}{\delta_T} > 0$ and $T_G(S,G) = -\frac{1}{\delta_T} < 0$. The latter assumes $\chi_G = 1$, that solar geoengineering G is indeed available.

Note that beginning with this equation, we drop time "(t)" for notational expediency and readability.

While $\dot{T} = 0$, per Assumption 1, S evolves according to a set of dynamic forces that require a look at the full optimization problem. The current value

Hamiltonian is given by

$$\mathscr{H} = U(E) + Y - pE - D(\tilde{T}, S, G) - C(R, G, A) + \mu_S \left(E - \chi_R R - \delta_S S\right),$$

where $\mu_S(t)$ is the co-state variable associated with the carbon cycle equation (3) and, per (2), $\tilde{T} = T(S, G) - \chi_A A$, which, in turn, via (7), results in:

$$\tilde{T}(S, G, A) = \frac{\lambda S - \chi_G G}{\delta_T} - \chi_A A. \tag{8}$$

The conditions for an optimal solution are given by:

$$\frac{\partial \mathscr{H}}{\partial E} = U'(E) - p + \mu_S = 0, \tag{9}$$
$$\frac{\partial \mathscr{H}}{\partial R} = -C_R(R, G, A) - \chi_R \mu_S = 0, \tag{10}$$
$$\frac{\partial \mathscr{H}}{\partial G} = -D_T(\tilde{T}, S, G)\tilde{T}_G(S, G, A) - D_G(\tilde{T}, S, G) - C_G(R, G, A) = 0, \tag{11}$$
$$\frac{\partial \mathscr{H}}{\partial A} = D_T(\tilde{T}, S, G)\tilde{T}_A(S, G, A) - C_A(R, G, A) = 0, \tag{12}$$
$$\frac{\partial \mathscr{H}}{\partial S} = -D_T(\tilde{T}, S, G)\tilde{T}_S(S, G, A) - D_S(\tilde{T}, S, G) - \delta_S \mu_S = \rho \mu_S - \dot{\mu}_S, \tag{13}$$

and the transversality condition,

$$\lim_{t \to \infty} e^{-\rho t} \mu_S S = 0. \tag{14}$$

Following Kamien and Schwartz (1981) and especially Weitzman (2009*a*), we can already say a lot about the optimal solution. For one, we can define the optimal CO_2 tax as:

$$\tau \equiv -\mu_S. \tag{15}$$

From (9) and (15) we further see immediately that the marginal utility derived from emitting CO_2 into the atmosphere should equal the marginal cost of extracting fossil fuels, p, plus the optimal CO$_2$ tax, τ:

$$U'(E) = p + \tau. \tag{16}$$

The optimal CO$_2$ tax, however, is only one of two possible interventions with direct implications on atmospheric CO$_2$ stocks.[22] Carbon geoengineering, too, is linked directly to the tax τ via μ_S in equation (10), assuming it is indeed available.We are, thus, ready for the first policy conclusion linking more than one instrument.

LEMMA 1: *The marginal cost of carbon geoengineering should equal the optimal CO_2 tax.*

PROOF:

The result follows readily from equations (10) and (15), resulting in: $C_R = \tau$, assuming $\chi = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$. □

The introduction of carbon geoengineering alone has expanded the "conventional wisdom" presented in the introduction, though only slightly. Assuming carbon geoengineering is available without any further binding restrictions, its optimal use is guaranteed by an optimal CO$_2$ tax alone. This is not the case as we go further down the climate system chain, setting $\chi = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, to include solar geoengineering and adaptation.

For interpreting the most general, analytic solution, we require one further assumption around damages and costs:

ASSUMPTION 2: *Damages, $D(\tilde{T}(t), S(t), G(t))$, and costs, $C(R(t), G(t), A(t))$, are separable in each of their respective elements.*

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[22] Note that the optimal CO$_2$ price is distinct from the social cost of carbon, SCC, even though the two often get conflated, and not merely because it should be the "SC-CO$_2$." The SCC is the marginal price of a ton of CO$_2$ given today's path (U.S. Government Interagency Working Group on Social Cost of Carbon, 2015). The SCC, thus, only equals the optimal CO$_2$ price, if one were to assume that today's path is optimal, a heroic assumption, to say the least. The interaction of carbon and solar geoengineering on the marginal (non-optimal) SCC is itself a potentially important, policy-relevant extension of this work. See, e.g., Kotchen (2016) for a framework that lends itself to this exploration.

While limiting in some regards, this assumption is eminently sensible to a first-order approximation. While both D and C are strictly increasing in each of their elements, the respective cross-derivatives ought to be second order. There may be political forces that link the availability of solar geoengineering, for example, to carbon geoengineering, and, thus, modify the marginal cost function with respect to each other, but even the sign of these interactions is unclear. Their magnitude can be assumed to be small.

PROPOSITION 1: *Assuming separability in D and C, the optimal CO_2 price provides incentives to reduce emissions and to increase carbon geoengineering to their respective optimal levels, but the tax has no immediate effect on the amount of solar geoengineering, nor on adaptation.*

PROOF:

From Lemma 1, we know that τ directly induces the optimal level of both E and R. That is not the case for either G or A. We can see that most immediately by the absence of μ_S in both (11) and (12). That alone concludes the proof, as the only effect of τ on either E or R goes through the atmospheric CO_2 stock S, which entails a significant time delay. $\square$

We can explore the implications of Proposition 1 further by rewriting (9) in terms of τ as a function of E, $\tau(E)$. It readily follows that $\tau'(E) < 0$, using the implicit function theorem. That result puts the direct link between E and τ in an even clearer light. In the short run, with T in equilibrium, E and τ are directly, inversely linked. Higher emissions implies a lower tax. Conversely, a higher tax goes hand-in-hand with lower emissions.

Having solved for $\tau(E)$, we again use (15), this time to rewrite (10) to find $R(E)$. It is now evident that $R'(E) < 0$, again using the implicit function theorem. Much like E and τ, E and R, too, are directly inversely related. Combining the two, it follows that higher taxes imply increased use of carbon geoengineering.

Exploring the implications for solar geoengineering, from (11) we find that the total marginal costs of solar geoengineering—marginal damages plus marginal costs of implementation—are equal to the marginal reduction in temperature-induced damages:

$$D_G(\tilde{T}, S, G) + C_G(R, G, A) = -D_T(\tilde{T}, S, G) T_G(S, G).$$

Finally, from (12), we learn that the marginal reduction in damages linked to temperature should be equal to the marginal costs of adaptation:

$$D_T(\tilde{T}, S, G) = C_A(R, G, A),$$

with $D_A < 0$. Combining equations (11) and (12), and assuming separability in costs and in damages from Assumption 2, we can derive the direct relationship between solar geoengineering and atmospheric CO_2 stocks, $G(S)$, and the same for adaptation, $A(S)$. Both are strictly increasing in S, with $G'(S) > 0$ and $A'(S) > 0$.

For the optimal evolution of the climate-economy system, taking S into account, we can take time derivatives of (9) to transform equation (13) into a dynamic equation that captures the evolution of emissions in the economy:

$$\dot{E} = (\rho + \delta_S)\frac{U'(E) - p}{U''(E)} - \frac{D_T(\tilde{T}, S, G) T_S(S, G) + D_S(\tilde{T}, S, G)}{U''(E)}. \tag{17}$$

Together with equation (3), transversality condition (14), and the additional boundary conditions $S(0) = S_0 > 0$, (17) captures the full dynamics of the system as it approaches the *slow equilibrium.*

B. ...and Slow

Per Assumption 1, temperature T is always in a steady state, what we have called the *fast equilibrium.* We learn more about the solution from analyzing the *slow equilibrium*, by setting both $\dot{E} = 0$ and $\dot{S} = 0$.

Setting (17) equal to zero, and replacing the implicit solutions for G and A, we find E as a function of S, $E_a(S)$, such that $E_a(0) = E_{BAU}$ and

$$E_a'(S) = \frac{dE_a}{dS} = \underbrace{D_{TT}(\tilde{T}, S, G)}_{\geq 0} \underbrace{\tilde{T}_S(S, G, A)}_{>0} \times \underbrace{\left(\frac{\tilde{T}_S(S, G, A) + \tilde{T}_G(S, G, A)G'(S) + \tilde{T}_A(S, G, A)A'(S)}{(\rho + \delta_S)U''(E)} \right)}_{<0} + \underbrace{\frac{D_{SS}(\tilde{T}, S, G)}{(\rho + \delta_S)U''(E)}}_{<0} < 0 \tag{18}$$

From this equation it follows that the slope becomes less negative when solar geoengineering and adaptation are part of the optimal policy, $\chi = \begin{pmatrix} \chi_R & 1 & 1 \end{pmatrix}$. See next section for an example that lends itself to simple graphical analysis. Setting $\dot{S}$ from Equation (3) equal to zero, and replacing $R(E)$ we find a function $E_b(S)$ such that $E_b(S) = E_{BAU}$ when $R = 0$ and $S = S_{BAU}$; and

$$E_b'(S) = \frac{dE_b}{dS} = \frac{\delta_S}{1 - \chi_R \underbrace{R'(E)}_{<0}} > 0. \tag{19}$$

It follows that the slope is less steep when carbon geoengineering R is available, $\chi = \begin{pmatrix} 1 & \chi_G & \chi_A \end{pmatrix}$. $E_b(S)$ with carbon geoengineering, thus, always lies above $E_b(S)$ without it for $S \in (0, S_{BAU})$. The combination of Equations (18) and (19) suggests that in a *slow equilibrium* with $\chi = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, emissions are lower and optimal CO_2 taxes are higher than if any or all of these additional policies are available:

PROPOSITION 2: *The introduction of carbon geoengineering, solar geoengineering, adaptation, or a combination thereof, increases emissions and reduces the optimal carbon tax.*

PROOF:

In the text above. □

While this particular Proposition is easily proven by analyzing Equations (18) and (19) directly, many other results are more easily interpreted when looking at graphical representations of how each climate policy affects the optimal outcome, and how they interact.

III. Optimal Climate Policy in Graphs

In what follows, we further explore the solution using simple graphs representing emissions E, stocks S, and temperatures T. To do so, we add a further structural assumption, making utility as well as cost and damage functions linear-quadratic:

ASSUMPTION 3: *Utility $U(E)$, costs $C(R,G,A)$, and damages $D(\tilde{T},S,G)$ are all assumed to be linear-quadratic in each element, such that:*

$$\begin{aligned} U(E) &= \alpha E - \beta E^2, \\ C(R,G,A) &= \frac{1}{2}\nu R^2 + \frac{1}{2}\eta G^2 + \frac{1}{2}\omega A^2, \mathit{and} \\ D(\tilde{T},S,G) &= \frac{1}{2}\kappa(\tilde{T})^2 + \frac{1}{2}\sigma S^2 + \frac{1}{2}\gamma G^2, \end{aligned}$$

with each parameter, α, β, ν, η, ω, κ, σ, and $\gamma > 0$.

This Assumption, while clearly adding detailed structure on our prior, more general discussion, does not take away from the conclusions we are able to draw from the following graphs. The linear-quadratic structure for $U(E)$, in particular, adds little that our prior, quasi-linear structure in Equation (1) did not already assume.

A. Steady-State Analysis

Using the functional forms from Assumption 3, we can plot the steady-state solution and how it changes as we introduce different instruments, as shown in Figures 1-4. The analytic derivations are shown in the Mathematical Appendix.

We start by comparing the optimal climate policy with only emissions reductions to the climate policy that also considers carbon geoengineering R (Figure 1).

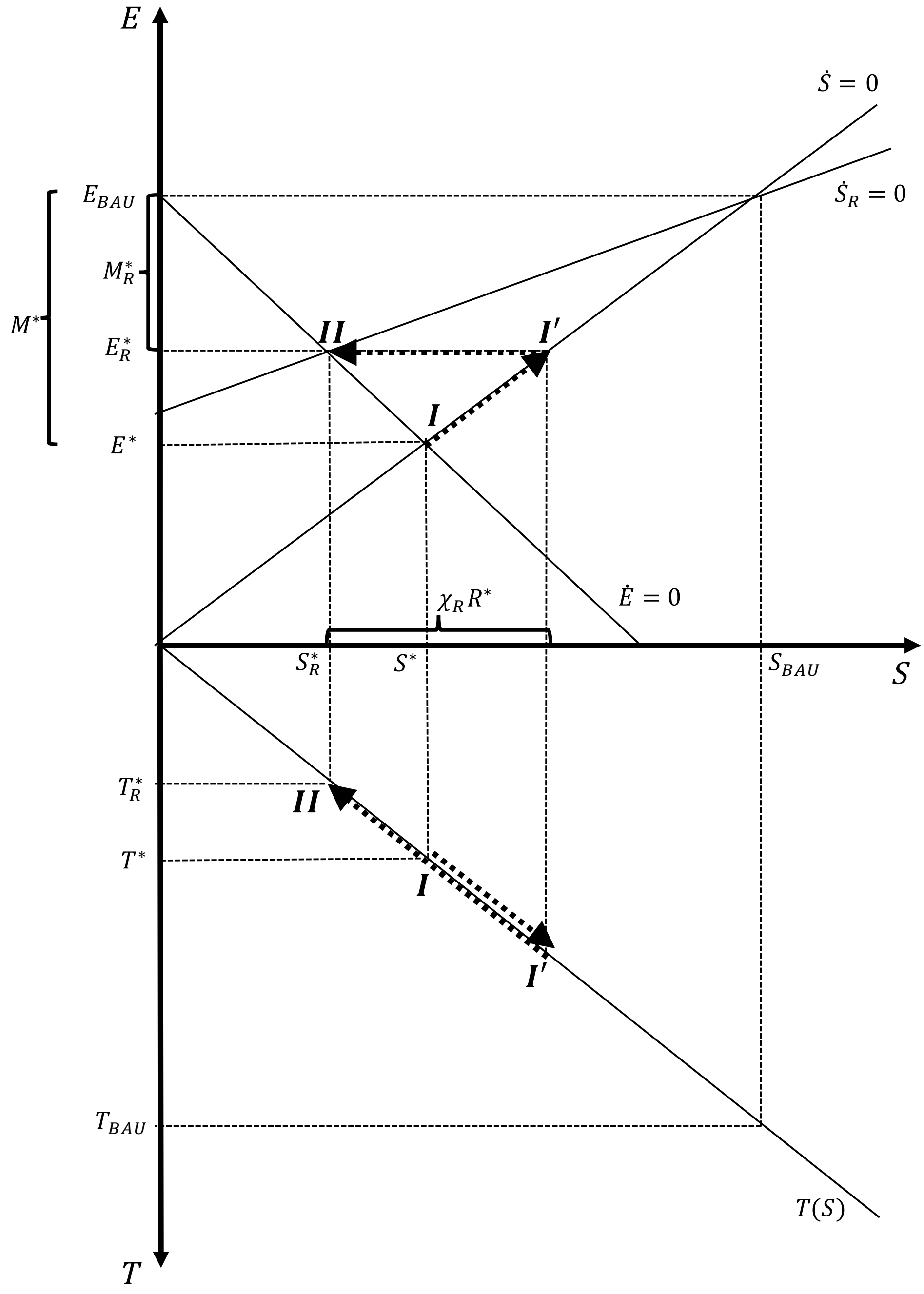

Figure 1. Optimal Climate Policy with Carbon Geoengineering

While the figure is ultimately what we hope to be an intuitive representation of optimal climate policy and its implications, encountering it for the first time merits some explanation. Figures 1-4 all share several characteristics. Most prominently, the figures have two panels each that share the same horizontal axis capturing atmospheric CO_2 stocks S. In the top panel we show the relation between emissions E and S, while in the bottom panel we show the relation between temperatures T and S. The bottom panel is flipped along the horizontal axis. Temperatures increase, as we move further away from the horizontal axis. Throughout this discussion, we make use of our derivations of $E_a'(S)$, Equation (18), and $E_b'(S)$, Equation (19).

Suppose first there is no need or desire for any climate policy interventions. Then we are at the business as usual (BAU) point (S_{BAU}, E_{BAU}) on the top right of the upper panel. This steady state is characterized by high emissions, high concentrations, and high temperatures. Welfare is correspondingly low. This scenario is clearly suboptimal.

There are four distinct possible climate policy interventions. We start with the case of emissions reductions alone, with $\chi = \begin{pmatrix}0 & 0 & 0\end{pmatrix}$. This steady state denoted by $I = (S^*, E^*, T^*)$ is characterized by the intersection between the lines $\dot{S} = 0$ and $\dot{E} = 0$. The mitigation-only steady state has lower emissions, concentrations, and temperature compared to the business as usual scenario. Figure 1 also calls out the amount of mitigation, given by the difference between business-as-usual and steady-state emissions $M^* = E_{BAU} - E^*$.

Next we introduce carbon geoengineering R, setting $\chi = \begin{pmatrix}1 & 0 & 0\end{pmatrix}$. Given Assumption 2 around separability of D and C, the introduction of carbon geoengineering only affects the $\dot{S} = 0$ equation that is now given by $\dot{S}_R = 0$. As discussed in the text leading up to Proposition 2, the new $E_b(S)$ line is flatter and lies above it for $S \in (0, S_{BAU})$, as it intersects with $\dot{S} = 0$ at (S_{BAU}, E_{BAU}). This change in the slope of the $E_b(S)$ line reflects a reduction in the marginal contribution of each unit of emissions to the accumulation of S in the atmosphere.

The steady-state equilibrium with mitigation and carbon geoengineering is, thus, given by $II = (S_R^*, E_R^*, T_R^*)$. Because mitigation and carbon geoengineering are substitutes in terms of their impact on stocks, $R'(E) < 0$, introducing carbon geoengineering necessarily results in higher emissions and lower mitigation. Those higher emissions would result in higher concentrations and higher temperatures, as denoted by the intermediate step I', where it not for the fact that the excess in concentrations is managed by the direct removal of carbon from the atmosphere by the amount of carbon geoengineering equal to R^*, moving us back to the steady state in II.

PROPOSITION 3: *Given Assumption 3, the introduction of carbon geoengineering G increases emissions E, reduces concentrations S and temperatures T, and overall increases welfare.*

PROOF:

See Figure 1 and the analysis above. See The Mathematical Appendix for the analytic proof. $\square$

We follow the basic structure of Figure 1 to introduce solar geoengineering and adaptation separately, followed by a general discussion of optimal climate policy, assuming $\chi = \begin{pmatrix}1 & 1 & 1\end{pmatrix}$.

For now we focus on $\chi = \begin{pmatrix}0 & 1 & 0\end{pmatrix}$: a closer look at the interaction of mitigation with solar geoengineering alone (Figure 2).

Introducing solar geoengineering, and only solar geoengineering, changes two elements in our figure: First, the steady-state relation between emissions and concentrations $E_a(S)$ rotates upwards, reflecting a reduction in the marginal damage of each unit of emissions. The second important change occurs in the bottom quadrant, where the relation between temperature and carbon concentrations changes from $T(S)$, to $T(S, G(S))$. This new line is flatter than the original reflecting a reduction in the marginal contribution of CO_2 to temperature changes.

With these changes in mind, we start again at the mitigation-only point, given by $I = (S^*, E^*, T^*)$. When solar geoengineering is introduced the new steady state

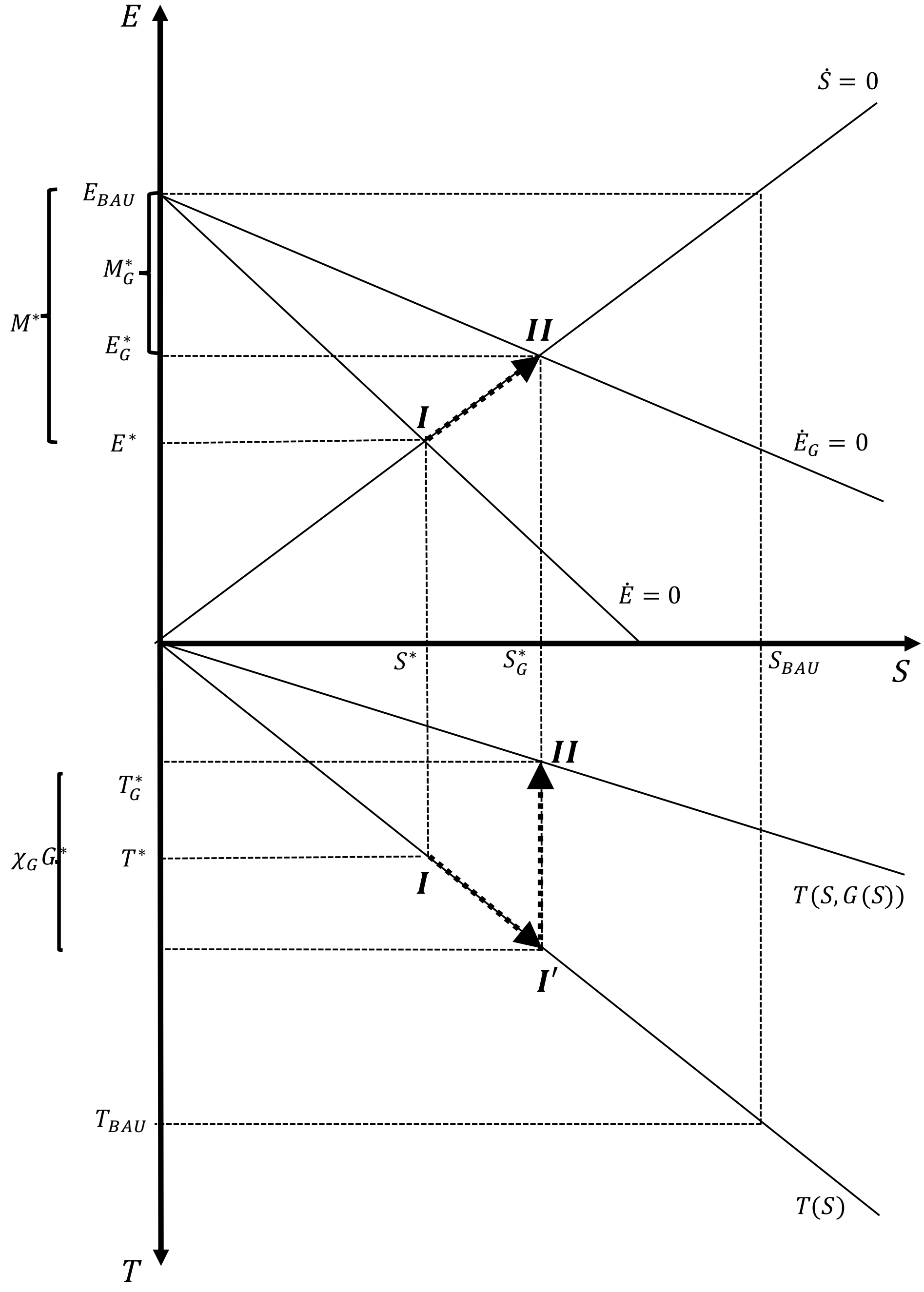

Figure 2. Optimal Climate Policy with Solar Geoengineering

moves to $II = (S^*_G, E^*_G, T^*_G)$. Because the introduction of solar geoengineering $G'(S) > 0$ reduces the marginal contribution of concentrations to temperature, both steady-state emissions and concentrations increase. Those higher emissions would result in higher temperatures, as denoted by the intermediate step I', but that excess warming is managed by the direct intervention of solar engineering equal to G^*, moving us down to the steady state temperature at point II.

PROPOSITION 4: *Given Assumption 3, the introduction of solar geoengineering G alone increases emissions E and concentrations S while reducing temperatures T, overall increasing welfare.*

PROOF:

See Figure 2 and the analysis above. See Mathematical Appendix for the analytic proof. □

One more missing element, before looking at all available climate policies in combination, is a quick look at adaptation alone, $\chi = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$. In some sense, we already know the outcome, as adaptation is a strictly private intervention, without addressing the root cause of excess S, nor lowering T. Worse, though, we show how adaptation leads to higher E, higher S, *and* higher T (Figure 3).

When adaptation is introduced, marginal damages from temperature are reduced, but it does not affect the relations between emissions and concentrations, nor between concentrations and temperature. That is, adaptation has a private, welfare-enhancing effect, without intervening in the global carbon-climate cycle. To see that most immediately, we start again at $I = (S^*, E^*, T^*)$, the mitigation-only steady state. With adaptation, the steady state immediately moves to $II = (S^*_A, E^*_A, T^*_A)$. Because the introduction of adaptation reduces the damages associated with any increase in concentrations, $A'(S) > 0$, there is an increase in the steady-state amount of emissions and concentrations. Those higher emissions result in higher temperatures. While the effects of that excess warming are managed by the direct intervention of adaptation equal to A^*, these changes

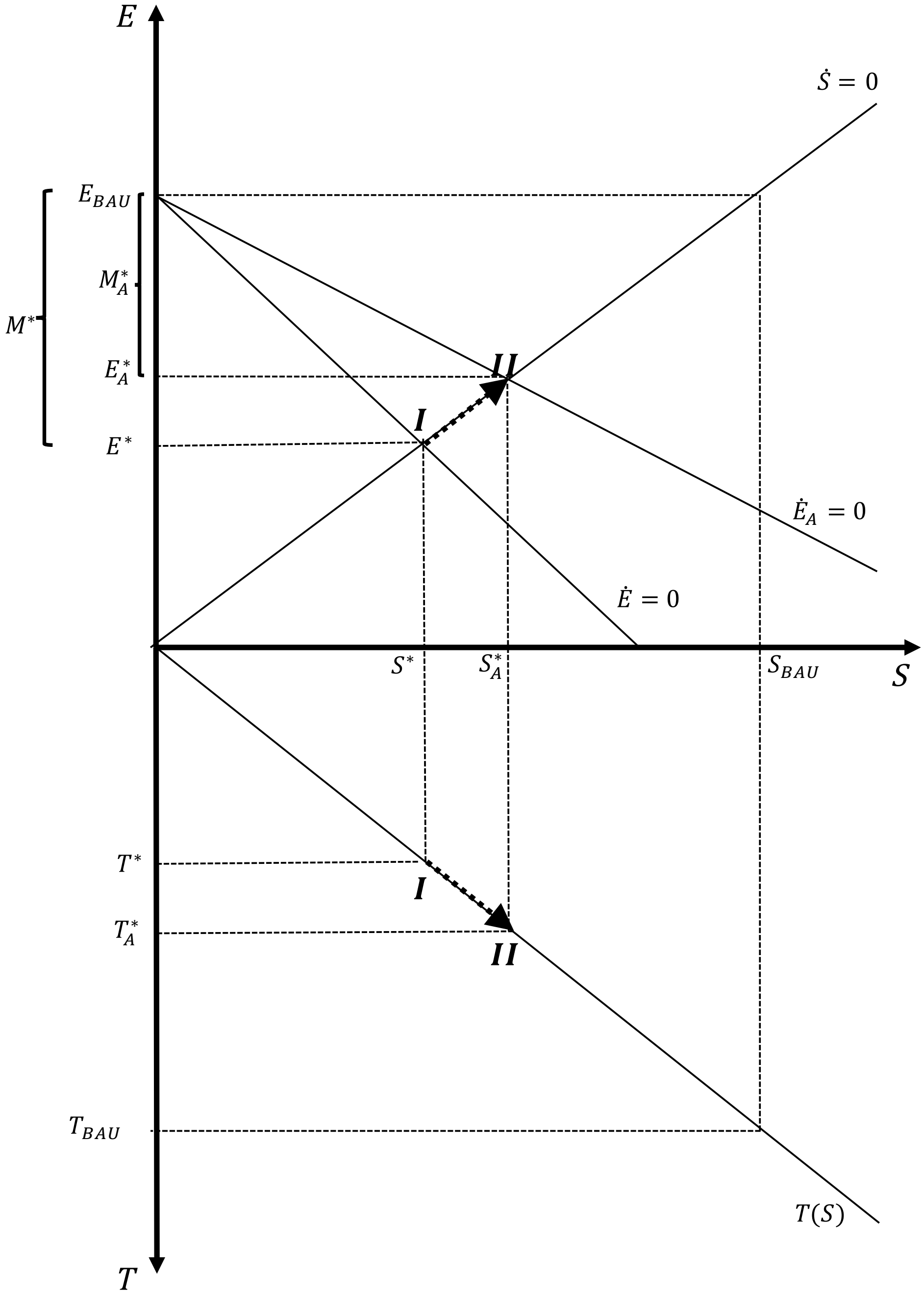

Figure 3. Optimal Climate Policy with Adaptation

are not shown in the figure for a simple reason: they are private, without effects on either global S levels or T.

PROPOSITION 5: *Given Assumption 3, the introduction of adaptation A increases emissions E, concentrations S, and temperatures T, while still increasing welfare due to a private reduction in damages.*

PROOF:

See Figure 3 and the analysis above. See Mathematical Appendix for the analytic proof. □

We can now turn to the analysis of the optimal climate policy when the full host of available interventions is introduced: $\chi = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$. See Figure 4.

First, recall that the introduction of adaptation and solar geoengineering results in higher emissions because it shifts up the $E_a(S)$ line but does not affect the placement of $E_b(S)$. This results in both higher emissions and concentrations. The introduction of carbon geoengineering increases emissions by shifting the $E_b(S)$ equation, but it also results in lower concentrations.

PROPOSITION 6: *Given Assumption 3, the introduction of carbon and solar geoengineering R and G as well as adaptation A increases emissions E, but it can increase or decrease temperatures T and concentrations S. Overall welfare increases.*

PROOF:

See Figure 4 and the analysis above. See Mathematical Appendix for the analytical proof.

Figure 4 depicts $S^*_{RGA} < S^*$. While both $E^*_{RGA} > E^*$ and $T^*_{RGA} < T^*$ is unambiguous, the decrease in S is not necessarily the case. Ultimately, the level of concentrations depends on the relative change in the slopes of $E_a(S)$ and $E_b(S)$.

What is unambiguous is that welfare increase when moving from the initial optimal state without any climate policies beyond mitigation, $I = (S^*, E^*, T^*)$,

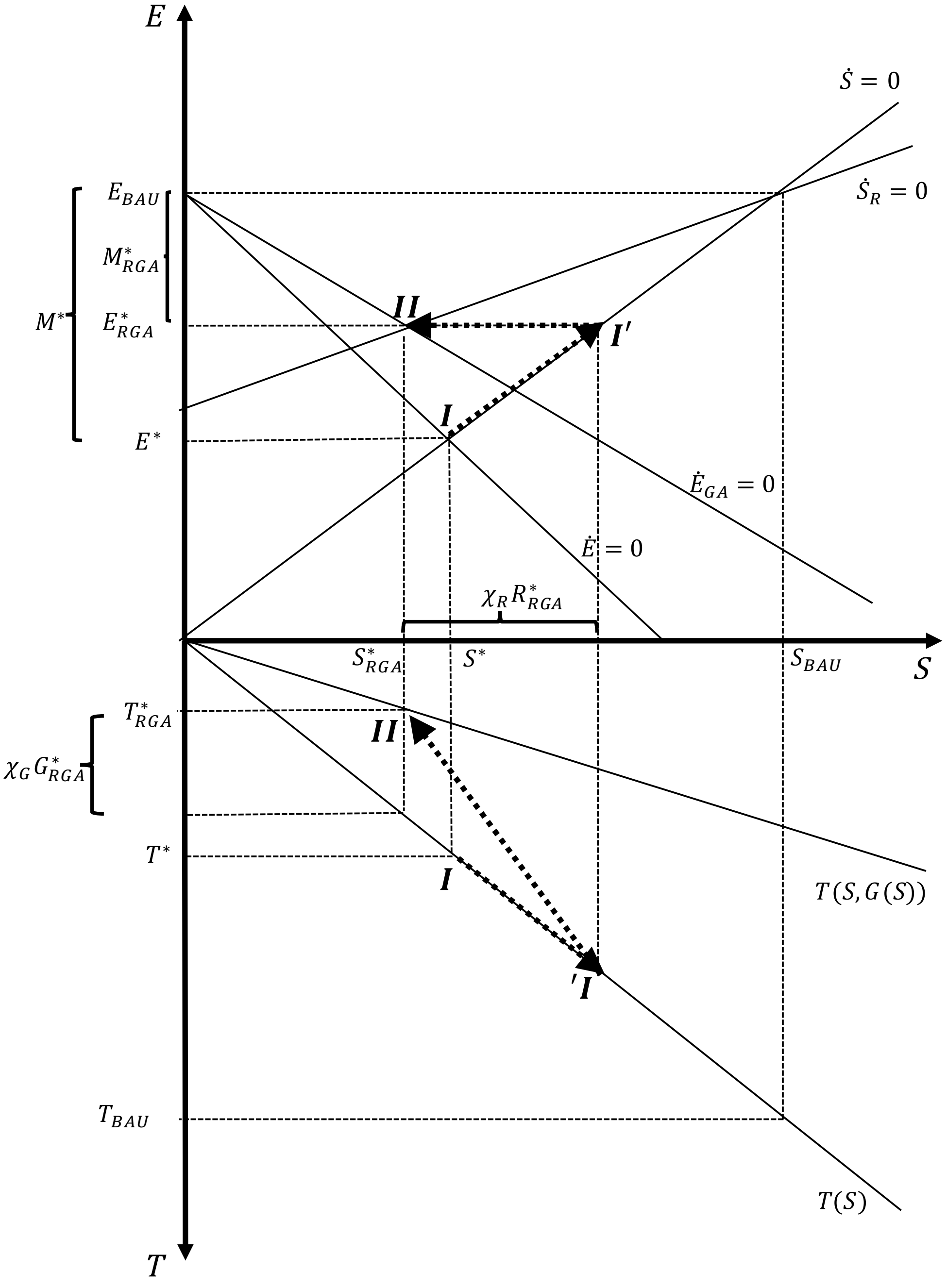

Figure 4. Optimal Climate Policy

via I' to $II = (S^*_{RGA}, E^*_{RGA}, T^*_{RGA})$. Expanding the set of available climate policy interventions increases societal welfare.

B. Phase diagram analysis

Up to now, we have been concerned with the steady-state behavior of the system, as we believe it conveys most of the information and nuances associated with the introduction of climate policies. In this section we present the phase-diagram analysis of the system to highlight the system dynamics and to discuss how the time path of climate policy changes as we introduce further options. We make use of Figure 5 to aid our discussion. The lines $E_a(S)$ and $E_b(S)$ divide the top quadrant in four sector. The arrows show the direction of movement in each sector. They follow from recognizing that

$$\frac{\partial \dot{E}(t)}{\partial S(t)} > 0 \quad \text{and} \quad \frac{\partial \dot{S}(t)}{\partial E(t)} > 0 \tag{20}$$

The dynamics follow four general rules, defined by the location relative to the $E_a(S)$ and $E_b(S)$ lines. For the two quadrants below the $E_a(S)$ line, the direction of motion is downward, for those above it is upward. For the two quadrants to the left of the $E_b(S)$ line, the direction of motion is rightward, for those to the right it is leftward. The overall system, thus, exhibits saddle path stability, with a stable arm reaching the unique steady state as shown in Figure 5.

Imagine two scenarios, one representing today, where concentrations are below the steady-state equilibrium. In that case, emissions should be falling as concentrations increase. Temperature, meanwhile, increases toward its corresponding (slow) equilibrium.

In the second scenario, concentrations are above the steady state, and it is time to bring them down. In this case, emissions drop immediately down from their starting level and then begin to increase, but concentrations are falling due to the combination of natural carbon decay and the use of carbon geoengineering.

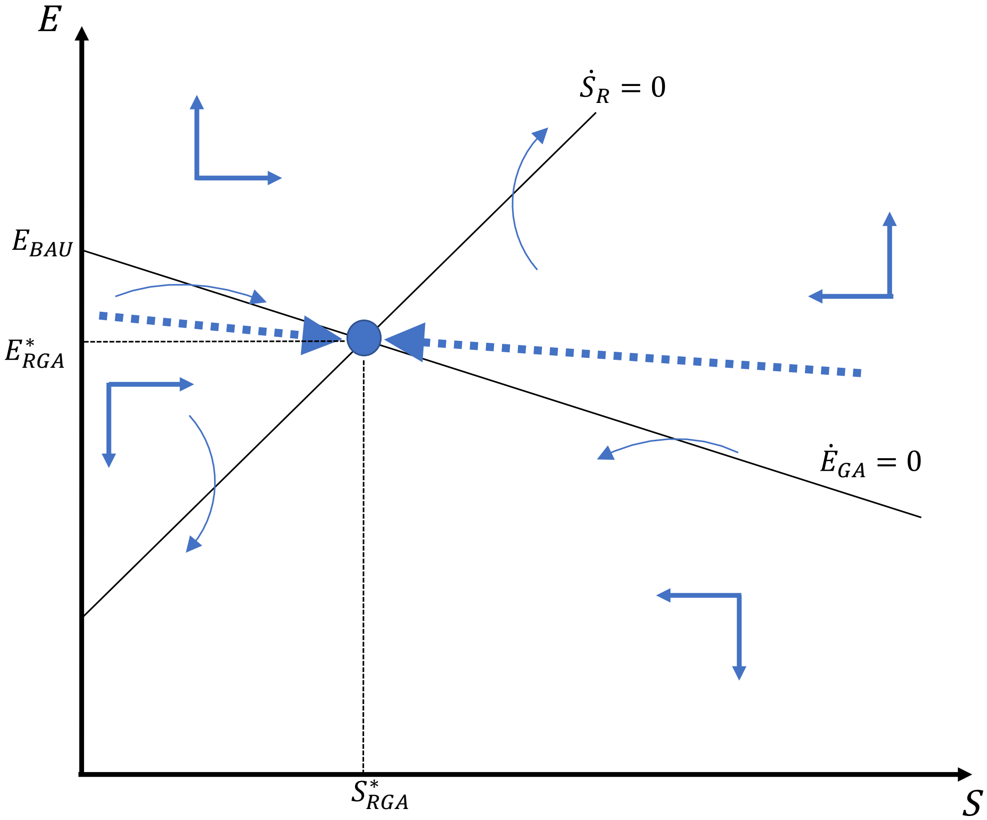

Figure 5. Optimal Solution Phase Diagram

Temperature is also falling as it reaches its steady state equilibrium.

Armed with this phase diagram, we are now able to see what happens when the system goes from a mitigation-only climate policy to one involving all four policies.

Let's start with a situation much like today, represented by Figure 6. Concentrations are lower than the steady state and are increasing, as the planet continues to warm. With only mitigation, the system eventually reaches a steady state given by (S^*, E^*). Suppose that at time t' all three policies—carbon and solar geoengineering as well as adaptation—are introduced. At this time, there will be a discrete jump in the emissions equal to $\Delta E(t')$. Concentrations will increase but at a lower pace because carbon geoengineering is now available. The introduction of solar geoengineering also allows for temperatures to jump at time t'. Lastly, damages from temperature fall immediately because of the use of adaptation. Thus, while the sudden introduction of these technologies would cause emissions to increase, overall it will reduce damages relative to a policy with only mitigation. Per Proposition 2, the associated optimal CO_2 price declines as well.

We could argue that climate policies beyond mitigation are even more relevant in a situation where the planet has already overshot both concentrations and temperatures beyond their (slow) equilibrium, needing to bring them down.[23] We show this scenario in Figure 7. The effects are clear: emissions increase, while carbon geoengineering ensures that concentrations are falling immediately. The atmospheric CO_2 stock falls smoothly and slowly, as it approaches the new steady state. As soon as solar geoengineering is introduced, temperatures jump instantaneously to a lower level. This characteristic is what makes solar geoengineering unique among climate policy interventions: it creates a jump in what would be a state variable, breaking the otherwise firm link between S and T.

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[23] Discussion of so-called "overshoot" scenarios has a long tradition in climate policy, going back at least to Broecker (2007).

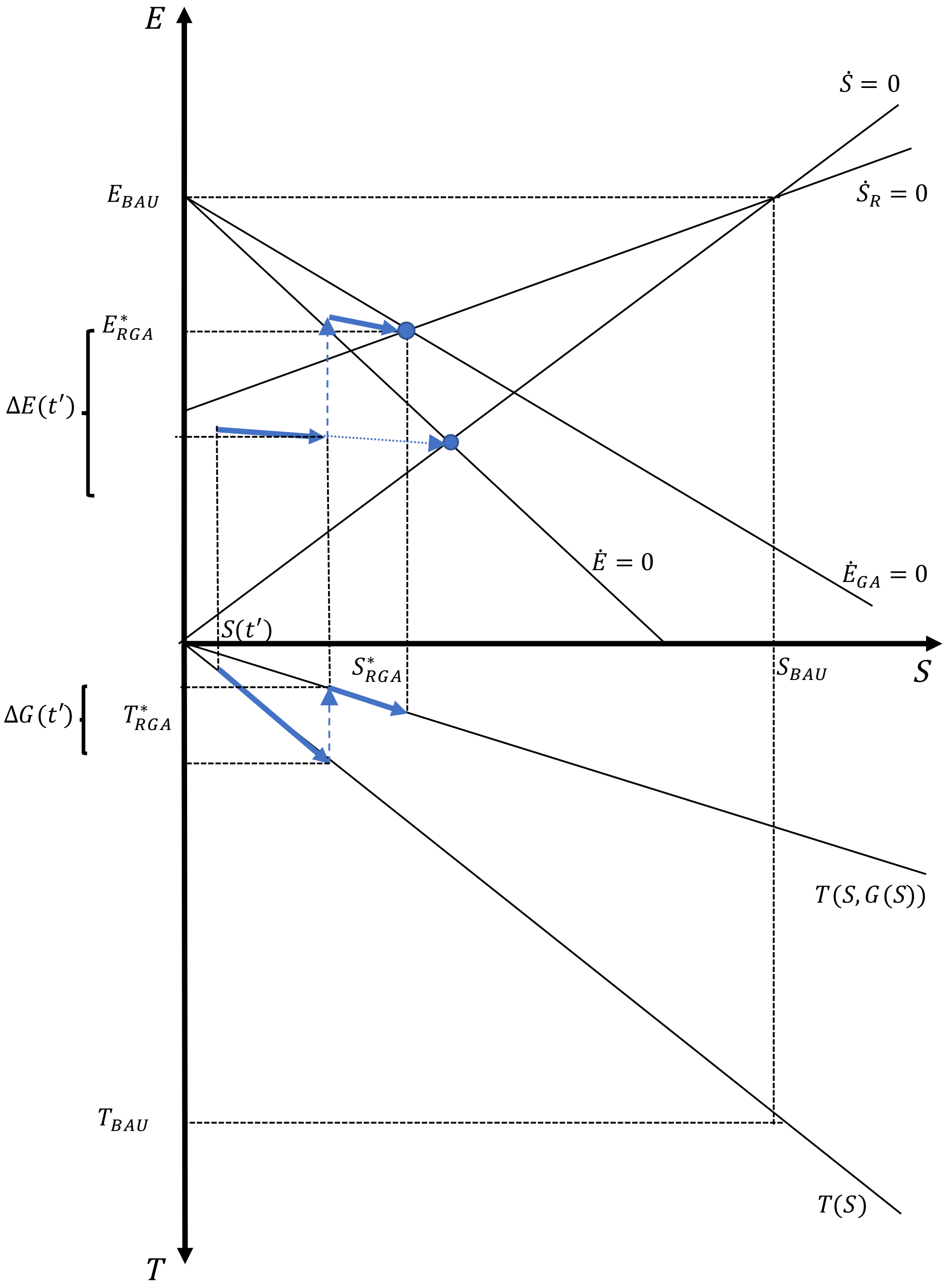

Figure 6. Sudden Introduction of Carbon and Solar Geoengineering from Low CO_2 Concentrations

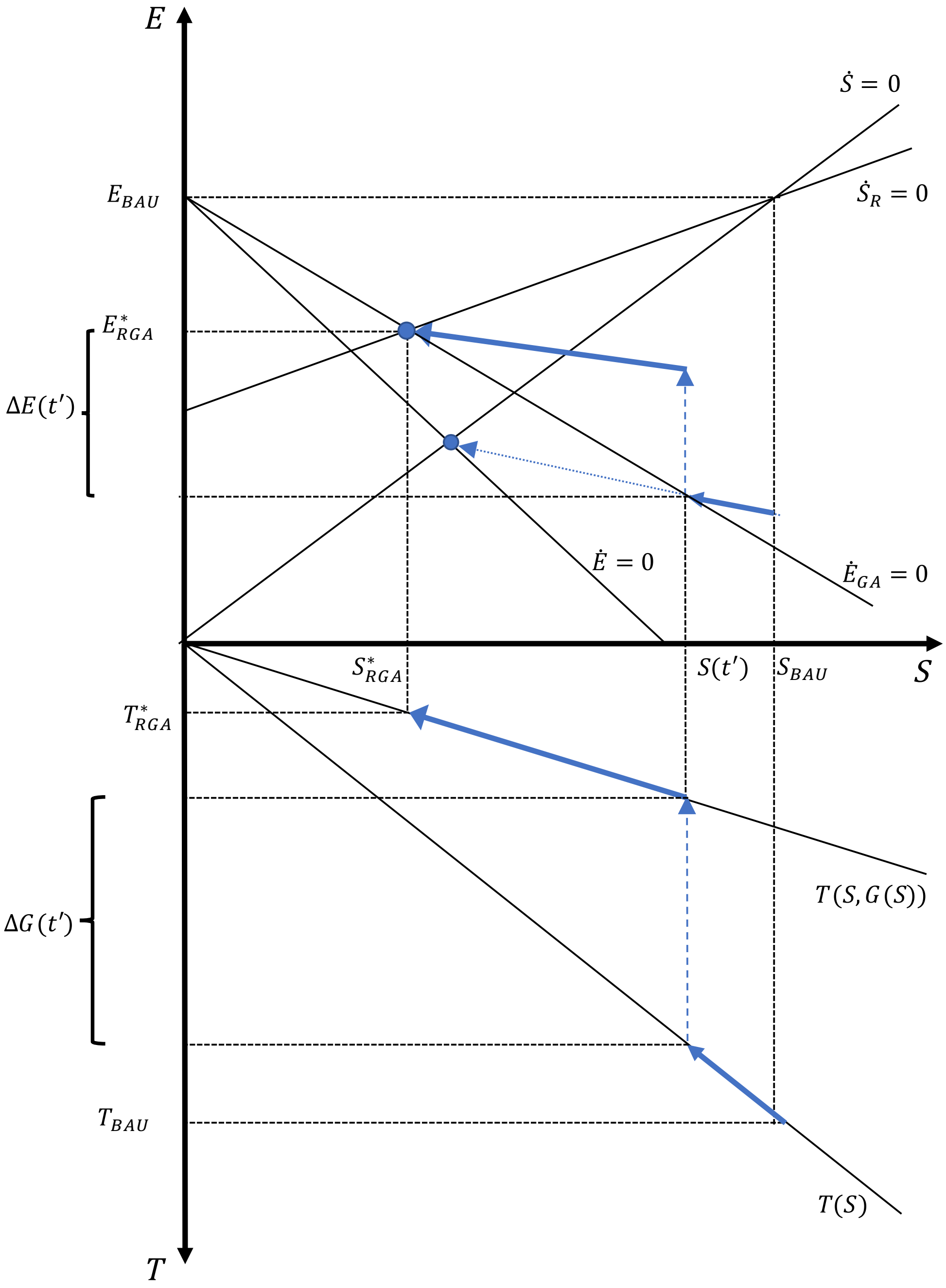

Figure 7. Sudden Introduction of Carbon and Solar Geoengineering from High CO_2 Concentrations

IV. Conclusions

This paper is at once easy and extremely difficult to summarize. It is easy to summarize because the main results are intuitive and supported by the canonical climate-economy model introduced here. It is difficult to summarize precisely because we attempt to introduce a basic taxonomy and canonical model that lends itself to exploring the most fundamental aspects of optimal climate policy.

The main contribution is reducing an incredibly complex problem to a set of two dynamic equations. While we could restate the main lessons and propositions here, the real contribution of this paper we believe is to add a simple framework—and simple graphs to go with that framework—to economic policy discussions to allow for a deeper exploration of the full set of climate policies: mitigation, carbon and solar geoengineering, and adaptation.

In doing so, we have introduced the concepts of *slow* and *fast equilibria*—one focused on the carbon cycle that takes decades, centuries or even millenia to approach its steady state, the other focused on temperatures, which are assumed to be in equilibrium at all times, or at least vis-à-vis the slow-moving carbon cycle. While this is surely a simplification of an otherwise complex climatic reality, it strikes us as an eminently sensible way to break down the problem without missing the main characteristics of the all-important emissions-concentrations-temperatures-damages chain on the one hand, and of the basic anatomy of climate policy interventions on the other. Ultimately, the main test of our model is precisely in what is currently missing: How do the main conclusions change as further characteristics of each policy intervention are introduced? How do the main conclusion change as the model itself is expanded to include multiple regions, or agents interacting with each other? How good a guide for actual climate policy is it?

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Mathematical Appendix

A1. Optimization problem and the optimal policy

The Hamiltonian of the problem, given Assumption 3, is:

$$\begin{aligned}\text{(A1)}\quad \mathscr{H} =&\alpha E - \frac{1}{2}\beta E^2 + Y - pE - \frac{1}{2}\kappa\left[\frac{\lambda S - \chi_G G}{\delta_T} - \chi_A A\right]^2 - \frac{1}{2}\sigma S^2 - \frac{1}{2}\gamma G^2 \\ &- \frac{1}{2}\nu R^2 - \frac{1}{2}\eta G^2 - \frac{1}{2}\omega A^2 \\ &+ \mu_s\left[E - \chi_R R - \delta_S S\right]\end{aligned}$$

The optimality conditions are:

$$\text{(A2)}\qquad \mathscr{H}_E = \alpha - \beta E - p + \mu_s = 0$$

$$\text{(A3)}\qquad \mathscr{H}_R = -\nu R - \chi_R \mu_S = 0$$

$$\text{(A4)}\qquad \mathscr{H}_G = \kappa\left[\frac{\lambda S - \chi_G G}{\delta_T} - \chi_A A\right]\frac{\chi_G}{\delta_T} - (\gamma + \eta)G = 0$$

$$\text{(A5)}\qquad \mathscr{H}_A = \kappa\left[\frac{\lambda S - \chi_G G}{\delta_T} - \chi_A A\right]\chi_A - \omega A = 0$$

$$\text{(A6)}\qquad \mathscr{H}_S = -\kappa\left[\frac{\lambda S - \chi_G G}{\delta_T} - \chi_A A\right]\frac{\lambda}{\delta_T} - \sigma S - \delta_S \mu_S = \rho\mu_S - \dot{\mu}_S$$

and the transversality condition is:

$$\lim_{t\to\infty} e^{-\rho t}\mu_S S = 0. \tag{A7}$$

We first look for expressions of each climate policy intervention beyond mitigation as a function of S, to be further able to express both $\dot{E}$ and $\dot{S}$ in terms of E, S, and parameters only. From equation (A5) we find:

$$A(S,G) = \frac{\chi_A}{\omega+\kappa\chi_A}\kappa T(S,G). \tag{A8}$$

Replacing (A8) back into (A4), in turn, allows us to find an expression for G as a function of S only:

$$G(S) = \frac{\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}{(\gamma+\eta)+\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}\lambda S. \tag{A9}$$

Replacing (A9) back into our expression for temperature in (7) we find

$$T(S,G(S)) = \frac{1}{\delta_T}\frac{(\gamma+\eta)}{(\gamma+\eta)+\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}\lambda S. \tag{A10}$$

From here we solve for A:

$$A(S) = \frac{\chi_A\kappa}{\omega+\kappa\chi_A}\frac{1}{\delta_T}\frac{(\gamma+\eta)}{(\gamma+\eta)+\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}\lambda S, \tag{A11}$$

which leads to an effective temperature expressed as:

$$\tilde{T}(S,G(S),A(S)) = \frac{1}{\delta_T}\frac{\left(\frac{\omega}{\omega+\kappa\chi_A}\right)(\gamma+\eta)}{(\gamma+\eta)+\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}\lambda S. \tag{A12}$$

Next, using equation (A2), we find $\mu_s = \beta E-(\alpha-p)$ and $\dot{\mu}_S = \beta\dot{E}$, and replacing

(A12) into (A6) we find:

$$\text{(A13)}\quad \dot{E} = \frac{1}{\beta}\left(\frac{\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)(\gamma+\eta)\frac{\lambda^2}{\delta_T^2}}{(\gamma+\eta)+\kappa\left(\frac{\omega}{\omega+\kappa\chi_A}\right)\frac{\chi_G}{\delta_T^2}}+\sigma\right)S+(\rho+\delta_S)E-(\rho+\delta_S)\left(\frac{\alpha-p}{\beta}\right).$$

Our next step is combine equations (A2) and (A3) to find:

$$\text{(A14)}\qquad R(E) = \frac{\chi_R}{\nu}(\alpha-p) - \frac{\beta\chi_R}{\nu}E,$$

which, in turn, allows us to find:

$$\text{(A15)}\qquad \dot{S} = \left(\frac{\nu+\beta\chi_R}{\nu}\right)E - \delta_S S - \frac{\chi_R}{\nu}(\alpha-p),$$

by replacing (A14) into the equation of motion for the stock of CO_2.

This concludes our initial derivations under the linear-quadratic Assumption 3. Equations (A13) and (A15), together with the boundary conditions $S(0) = S_0 > 0$ and transversality condition (A7) determine the dynamic behavior and return the optimal solution.

A2. Steady-state Analysis

The graphical analysis we present in the main text is based on this analytic results. Below are the proofs for all propositions that are not in the text. We begin by defining the different steady-state equilibria and then comparing the outcomes as we proceed with the analytic proofs.

ONLY MITIGATION, EQUILIBRIUM $I(S^*, E^*, T^*)$

In order to calculate the steady-state equilibrium with mitigation only, we set $\chi = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, resulting in:

$$S^* = \frac{E_{BAU}}{\delta_S + \phi_{BAU} + \psi_{BAU}}, \tag{A16}$$

$$E^* = \frac{\delta_S E_{BAU}}{\delta_S + \phi_{BAU} + \psi_{BAU}}, \text{ and} \tag{A17}$$

$$T^* = \frac{\lambda \delta_S}{\delta_T} \frac{E_{BAU}}{\delta_S + \phi_{BAU} + \psi_{BAU}}, \tag{A18}$$

where $E_{BAU} = \frac{\alpha - p}{\beta}$ are the business as usual emissions, $\phi_{BAU} = \frac{1}{\beta(\rho + \delta_S)} \kappa \frac{\lambda^2}{\delta^2}$ are the marginal effects of emissions on temperature damages, and $\psi_{BAU} = \frac{1}{\beta(\rho + \delta_S)} \sigma$ are the marginal effects of emissions on concentration damages.

INTRODUCING CARBON GEOENGINEERING, EQUILIBRIUM $II(S_R^*, E_R^*, T_R^*)$

In Figure 1 we introduce carbon geoengineering R. In order to calculate the steady-state equilibrium with only mitigation and carbon geoengineering we set $\chi = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$. Here we find:

$$S_R^* = \frac{\theta_R E_{BAU}}{\theta_R \delta_S + \phi_{BAU} + \psi_{BAU}}, \tag{A19}$$

$$E_R^* = (1 - \theta_R) E_{BAU} + \frac{\theta_R^2 \delta_S E_{BAU}}{\theta_R \delta_S + \phi_{BAU} + \psi_{BAU}}, \text{ and} \tag{A20}$$

$$T_R^* = \frac{\lambda \delta_S}{\delta_T} \frac{\theta_R E_{BAU}}{\delta_S + \phi_{BAU} + \psi_{BAU}}, \tag{A21}$$

where $\theta_R = \frac{\nu}{\nu + \beta} < 1$ is the marginal reduction in concentrations due to carbon geoengineering. This reduction in concentrations has two effects, as can be seen in the expression (A20) for E_R^*. First, each unit of emission is partially compensated by an increase in carbon geoengineering equal to $(1 - \theta_R)$. The remaining amount of carbon geoengineering works directly to reduce the stock of CO_2. This is also the main difference between mitigation and carbon geoengineering.

Proof Proposition 3: $S_R^* < S^*$ follows from observing that because $0\theta_R < 1$, the numerator in S_R^* is larger than the numerator in S^*. It follows directly that $T_R^* < T^*$. Simplifying Equation (A20) and realizing the numerator is smaller by a factor θ_R relative to the case with only mitigation leads to the conclusion that emissions increase. □

Introducing solar geoengineering, equilibrium II (S_G^*, E_G^*, T_G^*)

In Figure 2 we introduce solar geoengineering G, setting $\chi = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$:

$$S_G^* = \frac{E_{BAU}}{\delta_S + \phi_G \phi_{BAU} + \psi_{BAU}}, \tag{A22}$$

$$E_G^* = \frac{\delta_S E_{BAU}}{\delta_S + \phi_G \phi_{BAU} + \psi_{BAU}}, \text{ and} \tag{A23}$$

$$T_G^* = \frac{\lambda \delta_S}{\delta_T} \frac{\phi_G E_{BAU}}{\delta_S + \phi_G \phi_{BAU} + \psi_{BAU}}, \tag{A24}$$

where $\phi_G = \frac{(\gamma+\eta)}{(\gamma+\eta)+\frac{\kappa}{\delta_T^2}} < 1$ is the reduction in the effect of emissions on temperature damages. Solar geoengineering also modifies the temperature equation (A24), with ϕ_G entering the numerator.

Proof Proposition 4: From $\phi_G < 1$, it immediately follows that the factor reduces the denominators in both (A22) and (A23), thus increasing emissions and concentrations. In the temperature equation (A24), it reduces the numerator more than the denominator, thus reducing temperatures. □

Introducing adaptation, equilibrium $II(S_A^*, E_A^*, T_A^*)$

In Figure 3 we introduce adaptation A, setting $\chi = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$:

$$S_A^* = \frac{E_{BAU}}{\delta_S + \phi_A \phi_{BAU} + \psi_{BAU}}, \tag{A25}$$

$$E_A^* = \frac{\delta_S E_{BAU}}{\delta_S + \phi_A \phi_{BAU} + \psi_{BAU}}, \text{ and} \tag{A26}$$

$$T_A^* = \frac{\lambda \delta_S}{\delta_T} \frac{E_{BAU}}{\delta_S + \phi_A \phi_{BAU} + \psi_{BAU}}, \tag{A27}$$

where $\phi_A = \frac{\omega}{\omega+\kappa} < 1$ is the reduction in the effect of concentrations on temperature damages. Adaptation, however, does not modify the temperature equation (A27).

Proof Proposition 5: $\phi_A < 1$, reducing the magnitude of the denominator in all expressions, which increases concentrations (A25), emissions (A26), and temperatures (A27). □

It is worth noticing that, as expected, *effective* temperatures fall with adaptation:

$$\tilde{T}_A^* = \frac{\lambda \delta_S}{\delta_T} \frac{\phi_A E_{BAU}}{\delta_S + \phi_A \phi_{BAU} + \psi_{BAU}} < \tilde{T}^* (= T^*), \tag{A28}$$

with T^* from equation (A18).[24] This result follows from observing that ϕ_A appear both in the numerator and the denominator of $\tilde{T}_A^*$, but its effect in the numerator dominates the effect on the denominator. A lower effective temperature also implies lower damages D. Adaptation, thus, has a similar flavor to the case of solar geoengineering alone, but without global temperature effects.

Introducing all instrument, equilibrium $II(S_{RGA}^*, E_{RGA}^*, T_{RGA}^*)$

In Figure 4 we introduce all climate policy interventions at once, setting $\chi = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$:

$$S_{RGA}^* = \frac{\theta_R E_{BAU}}{\theta_R \delta_S + \phi_A \phi_{GA} \phi_{BAU} + \psi_{BAU}}, \tag{A29}$$

$$E_{RGA}^* = (1-\theta_R) E_{BAU} + \frac{\theta_R^2 \delta_S E_{BAU}}{\theta_R \delta_S + \phi_A \phi_{GA} \phi_{BAU} + \psi_{BAU}}, \text{ and} \tag{A30}$$

$$T_{RGA}^* = \frac{\lambda \delta_S}{\delta_T} \frac{\phi_{GA} \theta_R E_{BAU}}{\delta_S + \phi_A \phi_{BAU} + \psi_{BAU}}, \tag{A31}$$

where $\phi_{GA} = \frac{(\gamma+\eta)}{(\gamma+\delta)+\phi_A \frac{\kappa}{\delta^2}} > \phi_G$, reflecting the fact that adaptation and solar geoengineering are imperfect substitutes.

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[24] Without adaptation A, $\tilde{T}^* = T^*$.

Proof Proposition 6: $\phi_A < 1$, reducing the magnitude of the denominator in the emissions expression (A30). Overall impacts on T are undefined. $\square$

Following (A28), effective temperatures here, too, fall unequivocally:

$$\tilde{T}^*_{RGA} = \frac{\lambda \delta_S}{\delta_T} \frac{\phi_A \phi_{GA} E_{BAU}}{\theta_R \delta_S + \phi_A \phi_{GA} \phi_{BAU} + \psi_{BAU}} < \tilde{T}^*(= T^*), \tag{A32}$$

with T^* from equation (A18). Damages D fall as well.