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Steady-state growth in a Hotelling model of resource extraction

C.-Y. Cynthia Lin[a,*], Gernot Wagner[b]

[a]Agricultural and Resource Economics, University of California at Davis, One Shields Avenue, Davis, CA 95616, USA
[b]Harvard University, USA

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Abstract

This paper re-examines the Hotelling model of optimal nonrenewable resource extraction in light of stock effects and technological progress. We assume functional forms for cost and demand so that the solution to the Hotelling problem is a steady-state consistent with the empirical observation that the growth rates of market prices have remained zero over a long period of time. We use data on 14 minerals from 1970 to 2004 to estimate the supply and demand functions using SUR and 3SLS and to test the model. We validate the model for 8 of 14 minerals.

Keywords: Hotelling; Nonrenewable resources; Resource prices

1. Introduction

The basic Hotelling model of nonrenewable resource extraction predicts that the shadow price of a resource stock, which is equal to the market price minus marginal extraction cost and serves as an economic measure of resource scarcity, should grow at the rate of interest [8]. This prediction is commonly known as the "Hotelling rule."[1] Assuming constant marginal extraction costs and no technological progress, among other conditions, resource prices should be increasing over time. Real world prices do not follow this pattern, however. Empirical studies have shown that mineral prices have either been roughly trendless over time or have been stationary around deterministic trends with infrequent structural breaks.[2] This paper reconciles Hotelling's theoretical model with empirical evidence on world mineral prices.

The Hotelling rule may not be a good guide to the actual behavior of mineral prices over time for several reasons. In this paper we focus on two possible explanations: "stock effects" and technological progress.[3]

*Corresponding author. Fax: +1530 752 5614.
E-mail addresses: cclin@primal.ucdavis.edu (C.-Y. Cynthia Lin), gwagner@post.harvard.edu (G. Wagner).

[1]See, for example, Krautkraemer [9] for a recent survey of the literature on nonrenewable resource scarcity.

[2]See Krautkraemer [9] and references therein, Pindyck [17], and Lee et al. [10], as well as Lin [12,13] for the case of oil. Slade [20] has shown that mineral prices have followed a U-shaped price path between 1870 and 1970s, but the coefficients on the quadratic trend are not robust to the period of estimation [2]. From the 1970s through 2004, prices have been nonincreasing.

[3]In addition to stock effects and technological progress, the Hotelling model excludes several other factors. New discoveries, for example, expand the known stock of resource reserves and decrease the price through stock effects. Other factors include adjustment costs and capacity constraints due to the capital-intensive nature of mineral industries, market imperfections in the extractive industries, stock

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Stock effects increase extraction costs and are consistent with rising resource prices, while technological progress lowers extraction costs and causes prices to decline.[4] Slade [20] accounts for both of these effects in her theoretical model explaining U-shaped price trends. We expand Slade's theoretical model by estimating demand and supply functions and by allowing for a steady-state solution to the Hotelling model. We also expand the empirical analysis by using a more recent data set.

The simple Hotelling model assumes that extraction costs do not depend on the stock of reserve remaining in the ground, or equivalently, on the cumulative amount of resource already extracted. In fact, the original work by Hotelling [8] assumes that marginal extraction cost depends neither on the extraction rate nor on the remaining stock. Many researchers have followed this set of assumptions, while others have assumed that marginal extraction cost is an increasing function of extraction but independent of stock [22]. However, extraction costs will likely increase as more of the resource is extracted and fewer reserves remain [4].

Several reasons account for this phenomenon. Extraction costs rise if the resource is extracted from increasingly greater depths. For oil, greater extraction cost goes hand-in-hand with declining well pressure. Moreover, oil as well as other minerals come in different grades. Cheaper grades are likely to be mined before more expensive grades become economical, again leading to increasing extraction costs with cumulative stock extracted. Adding a stock effect to the classic Hotelling model causes shadow prices to rise less slowly than the rate of interest, but market prices still increase over time [11].

A second important reason for why the Hotelling rule may not adequately describe the actual behavior of world mineral prices is technological progress. Innovation enhances the extractive capacity of firms and decreases extraction costs over time.

In the next section, we combine stock effects and technological progress to develop a theoretical Hotelling model consistent with the often-cited stylized fact that mineral prices have remained constant over a long period of time. We then confirm this fact using data on 14 minerals over the period from 1970 to 2004. Section 3 estimates and tests the model's main proposition. We confirm the model's validity for 8 of 14 minerals. The final section concludes.

2. A theoretical model of resource extraction

2.1. The basic Hotelling model

In this section, we present a theoretical model of optimal nonrenewable extraction under perfect competition. We ignore inventories and assume competitive resource markets. We also ignore any common access problems that may arise in perfect competition.[5] Following Pindyck [16], we assume a large number of identical firms that act as price takers. This is equivalent to assuming that a social planner or a state-owned monopoly has sole production rights and sets a competitive price. The assumption of perfect competition may not be realistic for some nonrenewable resources such as oil, tin or nickel. Oligopolistic and monopolistic market structures may produce a bias towards conservation [19]. We abstract away from these problems in favor of tractability of our model.[6]

The notation follows closely that used by Weitzman [24]. Let $t \in [0, \infty]$ index time. At time t, the supply of the mineral is given by $E(t)$, the total extraction flow in tons per unit time at time t. Let $X(t)$ denote the total

(*footnote continued*)
effects on the benefit side caused by decreased environmental services due to resource extraction, and the ever present uncertainty. See Krautkraemer [9] for a further discussion of these factors.

[4]See also Farzin [4] for a general discussion of these effects.

[5]Sinn [19] presents a model of exhaustible resource extraction in which storage facilities are used as a result of the common-pool problem.

[6]Allowing for market power does not necessarily reconcile the theory with the data. For instance, according to Salant's [18] Nash–Cournot model with a cartel and a competitive fringe, the real price of oil increases monotonically for both constant and increasing marginal extraction costs. On the other hand, in Loury's [14] Cournot model, the present value of price net of the constant extraction costs declines over time. Moreover, minerals' markets that may seem to have market power may actually behave very much like perfectly competitive ones. For example, Agostini [1] finds that prices in the U.S. copper industry prior to 1978 were close to the levels predicted by a competitive model, even though the copper industry was highly concentrated.

cumulative stock of resource extracted at time t,

$$X(t) = X(0) + \int_0^t E(\tau)\, d\tau, \tag{1}$$

where the initial stock extracted $X(0)$ is taken as given. In contrast to Hotelling [8], no fixed quantity is assumed for the total availability of the resource. However, it is likely that only a limited total amount will be economically recoverable at any time [4], making this assumption more realistic.

The market price of the mineral at time t is $P(t)$. The corresponding demand is given by $D(P(t), t)$. Demand may shift over time; it may grow, for instance, owing to population growth or rising income, or it may shift downwards due to technological progress that enables consumers to use the resource more efficiently. At each time t, the market price $P(t)$ adjusts to equate supply and demand,

$$E(t) = D(P(t), t) \quad \forall t. \tag{2}$$

Total benefits $U(\cdot, \cdot)$ that accrue from the consumption of the mineral at time t are given by the area under the demand curve,

$$U(E(t), t) = \int_0^{E(t)} D^{-1}(x; t)\, dx, \tag{3}$$

where $D^{-1}(\cdot; t)$ is the inverse of the demand curve with respect to price. This area measures the gross consumer surplus and is a measure of the consumers' willingness-to-pay for the resource. Weitzman [24] shows that using the area under the demand curve in place of revenue yields the same outcome as a perfectly competitive market.[7] Thus, a perfectly competitive market maximizes total utility, or what Hotelling [8] terms the "social value of the resource". In the following discussion, therefore, we analyze the social planner's problem.

$C(X, E, t)$ depicts the cost of extracting E tons at time t. Solow and Wan [21] as well as Swierzbinski and Mendelsohn [23] discuss procedures for aggregating across multiple deposits of an exhaustible resource with different extraction costs. They show that in the absence of exploration, if firms extract first from the cheapest deposits and there are constant returns to scale in extraction, then an aggregate extraction cost function can be defined and indexed by the amount of cumulative extraction.[8] We use the term "stock effects" to refer to the dependence of extraction cost on the stock X of reserve extracted. This dependence will likely be positive.

Let $p(t)$ denote the nonnegative current-value shadow price measuring the value of a ton of reserve *in situ* at time t. This shadow price is known by a variety of terms, including marginal user cost, *in situ* value, scarcity rent, dynamic rent, and resource rent [3,9,24]. The competitive interest rate is r.

Net benefits $G(X, E, t)$ from extracting E tons at time t are given by total benefits minus total costs:

$$G(X, E, t) = U(E, t) - C(X, E, t). \tag{4}$$

The social planner's optimal control problem yields the same solution as would arise in perfect competition. The planner chooses her extraction profile $\{E(t)\}$ to maximize the present discounted value of the entire stream of net benefits, given initial stock $X(0)$ and the relationship between extraction $E(t)$ and cumulative stock extracted $X(t)$, and subject to the constraints that both extraction and stock are nonnegative. Her problem is

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[7]This holds because

$$P(t) = \frac{\partial U(E(t), t)}{\partial E},$$

so that the first-order conditions for the social planner's problem are the same as those that arise in perfect competition.

[8]Our extraction cost function in the next section exhibits constant returns to scale.

thus given by

$$\begin{aligned}\max_{\{E(t)\}} \quad & \int_0^\infty (U(E(t),t) - C(X(t),E(t),t))e^{-rt}\,dt \\ \text{s.t.} \quad & \dot{X}(t) = E(t) : q(t), \\ & E(t) \geqslant 0, \\ & X(0) = X_0, \end{aligned} \tag{5}$$

where $q(t) \leqslant 0$ is the co-state variable associated with the total extracted stock, $X(t)$. The absolute value of this variable is the shadow price $p(t)$ of the reserve still in the ground:

$$p(t) = |q(t)|. \tag{6}$$

Without stock effects ($\frac{\partial C}{\partial X}(\cdot,\cdot,\cdot) = 0$), the solution to the optimal control problem yields the Hotelling rule that the shadow price, price minus marginal cost, rises at the rate of interest,

$$\frac{\dot{p}(t)}{p(t)} = \frac{\frac{d}{dt}\left(P(t) - \frac{\partial C}{\partial E}(X(t),E(t),t)\right)}{P(t) - \frac{\partial C}{\partial E}(X(t),E(t),t)} = r. \tag{7}$$

If there are no stock effects and if marginal extraction costs are constant, then market prices increase over time. With zero marginal extraction costs, market prices grow at the rate of interest. With nonzero but constant marginal extraction costs, market prices still grow, but at a rate less than the interest rate.

We now examine what assumptions need to be made in order to obtain steady-state growth in the presence of stock effects and technological progress.

2.2. Steady-state growth

We now make functional form assumptions on both the demand function $D(P,t)$ and the cost function $C(X,E,t)$ so that the solution to the Hotelling resource problem (5) yields a constant growth rate in prices. In particular, we assume the cost function $C(X,E,t)$ is multiplicatively separable in each of its three arguments, and is of the form

$$C(X,E,t) = \Psi(X)Ee^{-\gamma t}, \tag{8}$$

and where the stock effect $\Psi(\cdot)$ is isoelastic,

$$\Psi(X) \equiv \Psi_0 X^b, \tag{9}$$

with a nonnegative elasticity $b \geqslant 0$. All else equal, a one percent increase in stock extracted causes costs to increase by b percent. Costs rise as more of the resource is being extracted. Marginal costs are therefore equal to average cost

$$\frac{\partial C(X,E,t)}{\partial E} = \frac{C(X,E,t)}{E} = \Psi(X)e^{-\gamma t} = \Psi_0 X^b e^{-\gamma t} \tag{10}$$

and are constant with respect to the extraction rate at any point in time.[9] Over time, cumulative extraction rises, which exerts upward pressure on costs. On the other hand, technological progress puts downward pressure on total and marginal costs. The stock effect and technological progress have countervailing effects on cost.

We assume that demand is isoelastic,

$$D(P,t) = AP^{-\frac{1}{\eta}}e^{\frac{\alpha}{\eta}t}, \tag{11}$$

[9]Heal [7], Hanson [5], and Solow and Wan [21] also assume that the extraction cost function exhibits constant returns to scale, where the marginal extraction cost is increasing in cumulative extraction but independent of the current rate of extraction.

where η is the inverse of the absolute value of the demand elasticity. Demand is growing over time if a is positive and decaying if a is negative.

With this demand curve, the total benefits $U(\cdot)$ from extraction at time t, as measured by the area under the demand curve, are given by

$$U(E, t) = \varphi(E)e^{at} \tag{12}$$

with $\varphi(\cdot)$ exhibiting constant elasticity,

$$\varphi(E) = A^{\eta}\frac{E^{1-\eta}}{1-\eta}. \tag{13}$$

From (3), the market price $P(t)$ satisfies:

$$P(t) = \varphi'(E(t))e^{at}. \tag{14}$$

The solution to the optimal control problem under these functional form assumptions for cost and demand leads to a linear relationship between extraction and stock. The proofs for the propositions are in Appendix A.

Proposition 1. *With isoelastic decaying costs* (8) *and isoelastic time-varying demand* (11), *the optimal extraction rate is given by*

$$E(t) = gX(t) \tag{15}$$

and the growth rate of prices is constant,

$$\frac{\dot{P}(t)}{P(t)} = -\eta g + a, \tag{16}$$

where the growth rate g is given by

$$g = \frac{\gamma + a}{b + \eta}. \tag{17}$$

The parameters satisfy

$$\lim_{t\to\infty}(A^{\eta}(gX_0)^{-\eta}e^{(-\eta g+a)t} - \Psi_0 X_0^b e^{(gb-\gamma)t})X_0 e^{(g-r)t} = 0 \tag{18}$$

and the coefficient Ψ_0 in the cost function is given by

$$\Psi_0 = \frac{(\eta g + r - a)A^{\eta}(gX_0)^{-\eta}}{(\gamma + r)X_0^b}. \tag{19}$$

In this model, a constant ratio of extraction to stock extracted implies a constant growth rate of prices.

Hartwick [6] also presents a model of steady-state in the presence of a nonrenewable resource. Under the Hartwick rule, the economy could sustain constant per capita consumption if it were to invest all the rents from the exhaustible resource in reproducible capital such as machines. The rents to be invested are precisely the Hotelling rents given by the Hotelling rule. The Hartwick rule therefore yields a steady-state in which consumption is constant and mineral prices are increasing. Our model differs from the Hartwick model both because it allows for constant mineral prices, which appear to better match the data than do increasing prices, and because there is the potential for constant growth in consumption at the steady-state.

Now that we have established that there is a set of parameters under which extraction rates and market prices grow continuously, we examine conditions under which the growth rates equal zero, as appears to be the case for many minerals.

Proposition 2. *The market price $P(t)$ is constant if the growth rate of extraction is given by*

$$g = \frac{a}{\eta}, \tag{20}$$

Table 1
Summary statistics for price and extraction data

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
Price (1982–1984 US$ per ton)														
Min	10.33	893.1	3.3E+6	19.19	23.97	357.6	1138.8	2869.8	33.49	20.66	5.08	8.7E+4	2249	452.8
Max	49.99	3797.8	2.6E+7	79.68	63.95	1627.8	4482.2	11517.8	327.74	120.76	21.09	8.8E+5	20841	2060.4
Mean	27.96	1910.5	9.8E+6	39.79	38.17	651.6	2103.2	6128.6	131.78	38.21	10.53	2.1E+5	8718	925.7
Std. dev.	12.12	760.0	4.4E+6	17.72	11.08	285.7	932.8	2104.5	72.76	22.94	4.69	1.6E+5	5734	332.1
World Extraction (Million tons)														
Min	153	16	0.0038	4290	1090	9	85	1	5140	199	719	0.025	0.58	14
Max	3580	331	0.0612	94700	20300	133	2020	49	99200	4560	25100	0.481	8.70	258
Mean	2125	199	0.0363	54164	12732	90	1104	28	61167	2806	14054	0.284	5.73	161
Std. dev.	995	88	0.0159	26237	5492	35	556	17	26644	1306	7851	0.132	2.27	69
Extraction by country (Million tons)														
Min	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Max	43.00	4.02	0.0026	1420.0	273.00	1.41	58.80	0.71	1370	90.20	579.0	0.0042	0.193	3.23
Mean	2.91	0.14	0.0000	41.54	8.60	0.07	0.78	0.03	36	3.76	19.7	0.0002	0.007	0.13
Std. dev.	6.14	0.38	0.0001	144.90	25.02	0.16	3.44	0.08	99	10.56	50.8	0.0005	0.017	0.29

BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc.
World extraction considers global extraction by year. Extraction by country includes pooled extraction rates for all years and all countries.
Source: Unpublished World Bank data.

or, *equivalently*,

$$\frac{\gamma}{b} = \frac{a}{\eta}. \tag{21}$$

The left-hand side of Eq. (21) captures technological progress and stock effects in the supply of minerals, the right-hand side captures changes in demand over time and the demand elasticity. For a balanced growth path, we need the two effects to equal each other. Thus, for market prices to be constant, the ratio of technological progress to the stock effect must exactly offset the exogenous growth in demand. Holding demand fixed, rapid technological progress, depicted by a large γ, needs to go hand-in-hand with large stock effects, b. Conversely, holding supply fixed, a higher exogenous growth rate a in the inverse demand function needs to be balanced by a more elastic demand, η, and vice versa. In reality all four coefficients will move simultaneously. This poses a challenge for the empirical analysis, which we will address in the following section.

3. Empirical analysis

We use extraction, price, and cost data for 14 subsoil assets from previously unpublished World Bank data. The commodities are oil, natural gas, brown and hard coal, as well as bauxite, copper, gold, iron, lead, nickel, phosphate, silver, and tin. The data cover 35 years from 1970 to 2004. Table 1 presents summary statistics. We supplement price data for some minerals with longer-term time trends from Berck and Roberts [2]. See Appendix B for a detailed description of the data.

Growth rates of prices are trendless and zero at the 5% significance levels for all commodities except for gold over the 35-year period from 1970 to 2004. The downward-sloping trend for gold is not surprising, considering that its price was indexed to the dollar through 1971 and it took several years for it to reach its natural equilibrium level. When we limit our data to the time period from 1974 to 2004, all 14 minerals, including gold, are trendless and zero. See Table 2 for the t-statistics of regressing the growth rate of prices on time and a constant. Fig. 1 shows absolute price levels and growth rates for gold for 1970–2004; Fig. 2 shows the same for oil for the extended time period 1870–2004.

Table 2
Tests of the stylized fact that the growth rates of prices equal zero

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
$\dot{P}(t)/P(t)$ 1970–2004														
Year	−1.04	0.71	−2.05**	−1.50	−0.54	−0.66	0.02	1.19	−0.91	−0.91	−1.50	−1.04	−1.29	−1.17
Constant	0.57	−0.72	1.95*	1.16	0.02	0.38	0.57	−0.58	1.10	0.78	1.16	0.95	0.71	0.93
$\dot{P}(t)/P(t)$ 1974–2004														
Year	−1.07	0.92	−1.00	−1.07	−0.37	−0.50	−0.18	1.20	−0.87	−0.86	−1.07	−0.81	−0.95	−0.30
Constant	0.65	−0.99	0.89	0.75	−0.13	0.24	0.78	−0.45	1.02	0.75	0.75	0.72	0.40	−0.04
$\dot{P}(t)/P(t)$ using Berck–Roberts data														
Year	1.00	0.07	—	—	0.06	0.17	1.53	—	0.31	—	—	1.10	—	−0.55
Constant	−0.98	0.25	—	—	0.23	0.19	−1.27	—	0.03	—	—	−0.96	—	0.98

The table presents t-statistics for regressions of the growth rates on year and a constant term.
BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc. Two stars (**) indicate significance at the 5% level, one star (*) indicates significance at the 10% level. Year in each case is normalized for the first year of the time period to equal 0. Regressions with robust standard errors. The third set of tests uses the entire data series from Berck and Roberts [2]. For BAU the years covered are 1895–1991, for COP 1870–1991, for IRO 1870–1985, for LEA 1870–1988, for OIL 1870–1991, for NG 1880–1991, for SIL 1901–1991, and for ZIN 1870–1991.
Source: Unpublished World Bank data and Berck and Roberts [2].

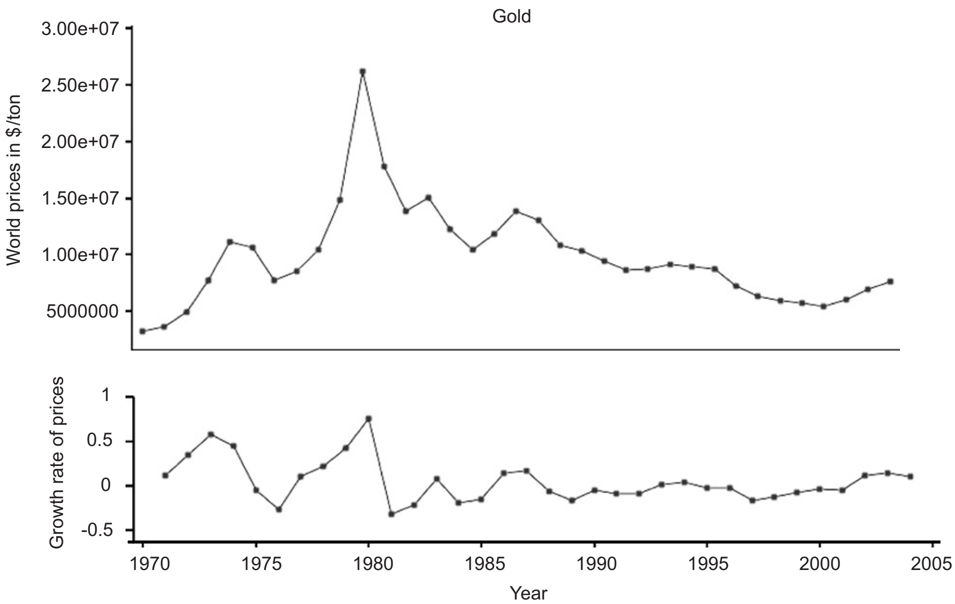

Fig. 1. World price and growth rates for gold from 1970 to 2004. (*Source*: Unpublished World Bank data. See Appendix B.)

Corresponding to Berck and Roberts's [2] result that prices are difference-stationary, we find that growth rates in prices are zero in all cases.[10] Lee et al. [10] use data similar to Berck and Roberts [2] to show that prices are stationary around deterministic trends with up to two structural breaks over the period 1870–1991. For all 11 minerals tested by Lee et al. [10], the last of these two breaks occurs between 1944 for bauxite and 1975 for natural gas, with the breaks for five minerals occurring between 1970 and 1975. These structural breaks correspond with relevant historical events. Our conclusions are unchanged, however, regardless of whether we

[10]These results are consistent with other prior studies ([9] and references therein; [12,13]).

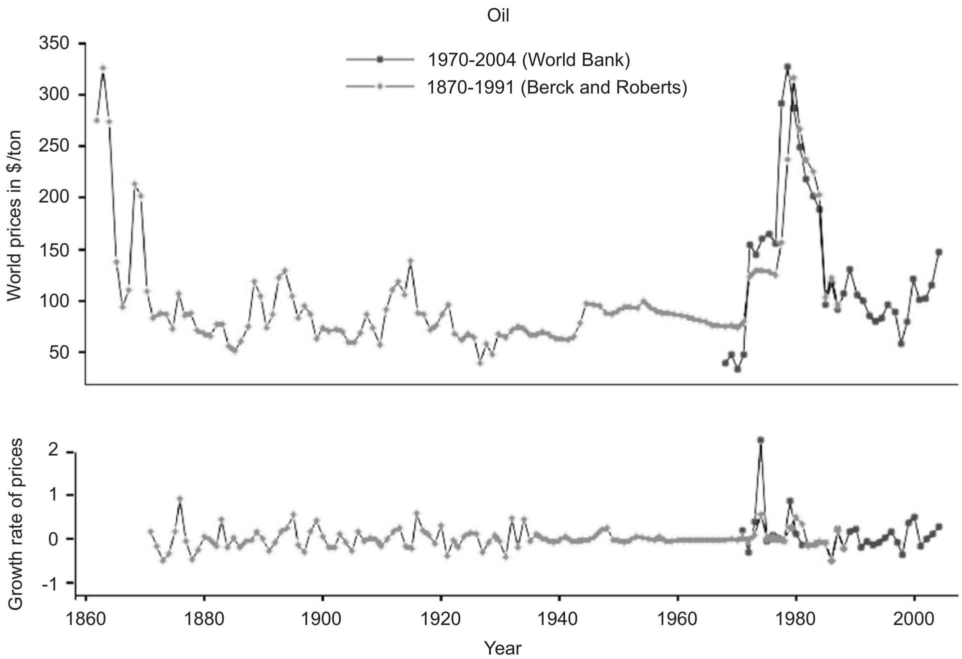

Fig. 2. World price and growth rates for oil from 1870 to 2004. (*Source*: Unpublished World Bank data and Berck and Roberts [2]. See Appendix B.)

use the entire series from 1970 to 2004 or limit ourselves to the period after 1974 or 1975. While Lee et al. [10] and studies cited by them find deterministic trends in their analyses from 1870 onwards, we do not find any such trends for the time between 1970 and 2004, the extent of our data set.[11]

Our main goal in this part of the paper is to estimate the theoretical model and verify its conclusions. Since our data show that, consistent with previous empirical studies, mineral prices have indeed been constant, we wish to see whether the data are consistent with our theoretical model, which generates a constant growth rate in prices and, under certain conditions, zero growth rates. We proceed in two steps. First we estimate the parameters of the model using the supply and demand functions given by Eqs. (8) and (11), respectively. We then proceed to test whether these parameters satisfy relationship (21) that Proposition 2 postulates should hold when the growth rate of prices is constant.

3.1. Estimation

We take the logarithms of Eqs. (10) and (11), resulting in

$$\ln MC = \ln AC = \ln \Psi_0 + b \ln X - \gamma t \tag{22}$$

and

$$\ln P = \ln A - \eta \ln E + at, \tag{23}$$

respectively. The data include values for average cost AC, extraction E, and price P. See Appendix B for a discussion of the data sources. Summing over E results in cumulative extraction X.[12] Given these data, we aim to estimate the stock effect coefficient $\ln \Psi_0$, stock elasticity b, and the rate of technological progress g in the supply equation, as well as the inverse absolute demand elasticity η, time trend a, and the constant demand shifter $\ln A$.

[11]Future research may identify another structural break in the period since 2004, given the recent rise in many natural resource prices. Our model may therefore be a description of the economy between structural breaks.

[12]We normalize $X(0)$ to be 0. See Appendix B.

We have no prior on the signs of the constant terms. Stock elasticity b has to be nonnegative to capture the expected stock effect. The rate of technological progress g should be positive, unless b fails to capture the full stock effect. Demand elasticity η needs to be nonnegative to avoid an upward-sloping demand curve. Demand may also shift across time. Positive a indicates an increase in demand, while a negative a would point to a decrease in demand for the particular mineral.

Tables 3 and 4 report results from jointly estimating supply and demand using the method of seemingly unrelated regression (SUR). Table 3 pools extraction across countries to arrive at world demand and supply

Table 3
Seemingly unrelated regression for demand and supply pooled across all countries

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
$\ln P =$														
$\ln A$	132.49	62.40	116.99	155.73	70.91	100.34	137.29	51.97	265.48	59.34	30.26	131.69	178.85	66.56
	(6.43)	(4.13)	(12.36)	(7.9)	(5.43)	(3.43)	(9.45)	(4.92)	(4.35)	(1.81)	(2.49)	(5.86)	(6.54)	(4.37)
η	0.83	−0.14	1.74	1.64	0.17	0.30	2.25	0.05	2.60	0.01	−0.53	0.83	0.48	0.16
	(3.14)	(0.71)	(12.16)	(5.28)	(1.27)	(1.32)	(12.23)	(0.69)	(4.67)	(0.03)	(3.35)	(2.47)	(2.06)	(1.10)
a	−0.06	−0.03	−0.04	−0.06	−0.03	−0.04	−0.05	−0.02	−0.10	−0.03	−0.02	−0.06	−0.08	−0.03
	(7.00)	(4.62)	(10.19)	(8.43)	(6.06)	(3.44)	(7.34)	(4.23)	(4.09)	(2.06)	(3.81)	(5.63)	(6.62)	(4.29)
$\ln \mathrm{MC} =$														
$\ln \varphi$	153.91	160.62	123.31	78.91	294.99	51.34	521.06	87.31	535.77	66.87	61.85	157.58	168.97	83.63
	(8.12)	(7.72)	(10.50)	(10.10)	(12.82)	(5.22)	(10.09)	(5.57)	(5.06)	(5.94)	(6.67)	(10.41)	(15.01)	(10.80)
b	0.29	0.07	0.92	0.16	0.94	−0.13	1.32	0.03	1.82	0.04	0.02	0.43	0.46	0.17
	(2.28)	(0.42)	(8.98)	(2.67)	(4.72)	(1.70)	(4.62)	(0.35)	(2.52)	(0.54)	(0.38)	(3.97)	(5.17)	(3.66)
γ	0.08	0.08	0.06	0.04	0.16	0.02	0.27	0.04	0.29	0.03	0.03	0.08	0.08	0.04
	(7.39)	(6.59)	(9.23)	(8.83)	(11.70)	(3.94)	(9.55)	(4.66)	(4.78)	(5.24)	(5.84)	(9.34)	(13.63)	(9.68)

BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc. Parentheses depict absolute values of t-statistics.
Source: Unpublished World Bank data.

Table 4
Seemingly unrelated regression for demand and supply with country fixed effects

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
$\ln P =$														
$\ln A$[a]	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—	—
η	0.01	−0.01	0.01	0.00	0.00	0.02	0.01	0.03	0.01	−0.03	−0.01	−0.01	−0.02	−0.02
	(1.77)	(1.91)	(2.53)	(0.81)	(1.16)	(1.79)	(1.50)	(3.34)	(1.09)	(2.75)	(1.20)	(2.24)	(3.92)	(4.04)
a	−0.04	−0.03	−0.01	−0.03	−0.03	−0.03	−0.01	−0.02	−0.01	−0.03	−0.03	−0.03	−0.06	−0.02
	(44.06)	(55.9)	(7.37)	(43.82)	(59.62)	(31.12)	(8.15)	(13.70)	(4.90)	(23.31)	(33.52)	(31.01)	(39.11)	(28.81)
$\ln \mathrm{MC} =$														
$\ln \varphi^{\ddagger}$	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b	0.01	0.01	0.01	0.01	0.01	−0.08	−0.01	−0.02	0.06	0.08	0.04	0.08	0.10	0.10
	(1.00)	(1.32)	(1.34)	(4.41)	(2.67)	(7.45)	(1.09)	(3.43)	(10.77)	(6.84)	(8.23)	(7.09)	(8.76)	(12.61)
γ	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.04	0.06	0.02
	(4.66)	(28.32)	(0.70)	(22.92)	(58.00)	(10.13)	(10.69)	(11.56)	(9.70)	(8.96)	(26.50)	(34.04)	(46.13)	(28.58)

BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc. Parentheses depict absolute values of t-statistics.
Source: Unpublished World Bank data.
[a]Constant terms in the demand and supply regressions vary by country and are not reported here. All are positive and significant at the 5% significance level.

Table 5
Economies with the highest ratio of extraction value to GDP by mineral

BAU	COP	GOL	HC	IRO	LEA	NG
Guyana Jamaica Suriname	Zambia Zaire Papua N Guinea	Guyana Papua N Guinea South Africa	Poland South Africa	Liberia Mauritania	Australia Namibia Peru Macedonia	Bahrain Romania Qatar
NIC	**OIL**	**PHO**	**SC**	**SIL**	**TIN**	**ZIN**
Botswana	Oman Qatar Saudi Arabia UAE Kuwait	Togo Jordan Morocco	Mongolia Romania Bulgaria	Peru Bolivia Honduras	Bolivia Nepal	Peru Zambia Bolivia

Source: Unpublished World Bank data.

for each of the 14 minerals. Table 4 includes country fixed effects, using vastly more information by relying on each country's extraction rates and accumulated stock.

The estimation results are similar. In the pooled regression, all but one of the estimates for η is nonnegative. Copper, iron, lead, nickel, phosphate, and zinc have inelastic demand; bauxite, gold, hard coal, natural gas, oil, silver, and tin have decreasing demand. The sole exception is brown coal, whose estimate for η is negative. *Ceteris paribus* the demand curve for all commodities, including oil and natural gas, seems to be shifting slightly downwards over time, captured by negative values of a in both pooled and fixed effects SUR. The downward shift in demand may be a result of technological progress such as energy-efficient technology that enables consumers to use the resource more efficiently. Stock elasticity b is nonnegative for all minerals in the pooled SUR. The stock effect for oil is by far the largest, followed by natural gas, iron and gold. A one percent increase in b results in a 1.8 percentage change in the marginal cost of oil extraction. The corresponding value for natural gas is 1.32. For iron it is 0.94, and for gold it is 0.92. The magnitude of b, however, decreases significantly once we account for country fixed effects. The coefficients for nickel and lead even turn negative, indicating that technological progress might not be fully captured by γ, or that we do not account for all other factors such as increasing reserves of the resource in the ground. We do find positive γ for almost all industries, indicating positive technological progress. The sole exception is gold under the specification of country fixed effects.

While SUR is more efficient than OLS, it is not necessarily consistent [15,25]. Demand and supply are simultaneously determined. To address this last point, we attempt to identify supply and demand shifters and employ three-stage least squares (3SLS) regressions. We identify aggregate world GDP as a demand shifter and use it as an instrument for supply. As an instrument for demand, we focus on small economies, for which production of a particular resource accounts for a large part of that nation's GDP. Assuming that domestic demand for the minerals is negligible compared to world demand, we can use that country's GDP as a supply shifter and, thus, as an instrument for demand. We identify appropriate countries by choosing the three countries for each mineral, which have the highest ratio of extraction value to GDP. These countries are listed in Table 5.

Table 6 presents the results for the 3SLS regression. The results are similar to the SUR regressions, both in sign and magnitude. We are unable to perform the same analysis using country fixed effects, since we use individual countries' GDP as instruments, making them no longer exogenous. Since 3SLS requires us to focus on world averages pooled across countries, and the results do not seem to be appreciably different from SUR estimates, our preferred specification is to use SUR with country fixed effects.

3.2. Test of model

Proposition 2 postulates that the parameters of the model will satisfy Eq. (21) if growth rates of prices are zero. Using the coefficients from SUR with country fixed effects presented in Table 4, we proceed to test

Table 6
3SLS regression for demand and supply with World GDP as demand shifter and small exporting countries' GDP as supply shifter

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
$\ln P =$														
$\ln A$	117.0	130.57	134.05	235.65	135.72	19.57	0.00	32.61	0.00	0.00	14.31	572.74	158.42	112.32
	(11.8)	(3.57)	(9.2)	(5.07)	(2.26)	(0.23)	(0.00)	(1.12)	(0.00)	(0.00)	(0.76)	(2.84)	(2.35)	(1.21)
η	0.09	1.03	2.19	2.79	0.83	0.20	−0.02	−0.34	−0.09	−0.63	−0.93	7.64	0.30	0.72
	(0.59)	(1.95)	(5.31)	(3.89)	(1.36)	(0.12)	(0.07)	(0.93)	(0.13)	(4.18)	(2.9)	(2.46)	(0.53)	(0.76)
a	−0.06	−0.05	−0.05	−0.09	−0.06	−0.01	0.00	−0.01	0.00	0.00	−0.02	−0.25	−0.07	−0.05
	(14.8)	(3.72)	(8.33)	(5.45)	(2.42)	(0.17)	(1.34)	(1.14)	(0.19)	(2.92)	(2.2)	(2.83)	(2.41)	(1.2)
$\ln \mathrm{MC} =$														
$\ln \varphi$	173.8	234.72	271.11	109.67	209.98	0.00	609.77	84.63	713.71	78.95	108.96	261.39	199.49	90.68
	(2.63)	(4.27)	(4.83)	(9.08)	(11.95)	(0.00)	(8.17)	(4.62)	(4.62)	(4.75)	(6.97)	(8.6)	(10.77)	(7.36)
b	0.47	1.53	2.88	0.37	0.44	−0.86	2.03	0.04	3.72	0.12	0.28	1.25	0.76	0.31
	(0.5)	(1.73)	(3.62)	(4.05)	(3.09)	(0.52)	(3.78)	(0.38)	(2.65)	(1.22)	(3.07)	(5.5)	(4.79)	(3.41)
γ	0.09	0.13	0.14	0.06	0.11	−0.01	0.33	0.04	0.40	0.04	0.06	0.13	0.10	0.05
	(2.11)	(3.59)	(4.43)	(8.19)	(10.61)	(0.71)	(7.59)	(3.88)	(4.3)	(4.25)	(6.45)	(8.03)	(9.79)	(6.54)
$\ln E =$														
GDP_X	1.91	−0.38	−0.38	−43.82	−9.99	−5.60	−0.02	0.60	0.06	−0.07	0.20	0.00	2.25	0.11
	(3.21)	(1.00)	(0.15)	(0.58)	(0.75)	(1.12)	(3.96)	(0.9)	(4.72)	(5.14)	(1.51)	(0.01)	(5.66)	(1.66)
$\ln X =$														
GDP_W	2.05	2.30	0.08	9.39	7.40	−0.12	17.99	25.37	1.20	19.95	61.88	5.95	7.28	15.33
	(3.78)	(5.75)	(0.92)	(5.46)	(8.98)	(0.73)	(2.59)	(2.25)	(0.99)	(5.35)	(3.34)	(2.24)	(2.35)	(1.68)

BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc. Parentheses depict absolute values of t-statistics.
World GDP, GDP_W, and the GDP of small exporters, GDP_X, are in log values. The three small exporting countries used in the instrumental variable regression for demand are the three countries with largest share of mineral production relative to GDP.
Source: Unpublished World Bank data.

Table 7
Test of main result in Proposition 2 using coefficients from SUR with country fixed effects

	BAU	COP	GOL	HC	IRO	LEA	NG	NIC	OIL	PHO	SC	SIL	TIN	ZIN
$\mathrm{H}_0 : \frac{\gamma}{b} = \frac{a}{\eta}$														
$\chi^2 =$	4.51	0.45	5.14	0.78	2.10	3.13	0.25	0.31	1.92	5.13	1.09	2.95	8.46	9.04
Prob. $> \chi^2 =$	0.03	0.50	0.02	0.38	0.15	0.08	0.61	0.57	0.17	0.02	0.30	0.09	0.00	0.00

BAU, bauxite; COP, copper; GOL, gold; HC, hard coal; IRO, iron; LEA, lead; NG, natural gas; NIC, nickel; OIL, oil; PHO, phosphate; SC, brown coal; SIL, silver; TIN, tin; ZIN, zinc.
The critical value of χ^2 at the 5% significance level is 3.84. At the 1% significance level, it is 6.63. For values above that value, we reject the null hypothesis. The second row depicts the probability of not rejecting the null hypothesis. For bauxite (BAU), for example, we reject the null hypothesis at the 5% significance level because 0.03 <0.05; we do not reject it at the 1% significance level because 0.03 >0.01.
Source: Unpublished World Bank data.

Eq. (21) using a Wald test. The null hypothesis states that $\frac{\gamma}{b} = \frac{a}{\eta}$. The critical χ^2 value at the 5% significance level equals 3.84. We hope not to reject the null to verify the validity of our model. This is the case for 9 of 14 minerals at the 5% significance level. Table 7 presents the test results.[13]

[13]The nonlinear Wald test is not consistent across different, mathematically equivalent, specifications. Rewriting the null hypothesis as $\gamma = \frac{a}{\eta} b$, for example, would result in fewer rejections. Stating the null hypothesis in terms of $\frac{\gamma}{b} = \frac{a}{\eta}$ provides the more restrictive test of our model.

Three of the minerals where the test fails under any significance level are phosphate, tin and zinc. This is not surprising, however, since all three minerals also have negative estimates for η. In that light, we should also exclude silver, since its η is negative as well. This leaves 8 of 14 minerals, for which the model yields appropriate coefficients and its main conclusion is shown to hold.

4. Conclusion

We modify the standard Hotelling model of natural resource extraction by accounting for stock effects and technological progress. This enables us to achieve a balanced growth path and estimate demand and supply functions for minerals. We then estimate the model using data from 1970 to 2004. The prices for 13 of 14 minerals exhibited a zero growth rate over that 35-year period. Using the estimated coefficients, we test the model and validate it for 8 of 14 minerals.

An important caveat to note is that our model predicts ever growing economically recoverable stocks and ever growing extraction. On a finite planet, this is clearly not possible forever. Our model does point to a larger truth, though. Most resource prices have not increased between 1970 and 2004, the years covered by our data. We argue that this was due largely because technological innovation in both the supply side and the demand side was able to offset stock effects. Since 2004, however, prices for many subsoil assets have increased sharply. One possible explanation for this increase is that it is only a temporary spike that will soon be followed by a decrease in prices driven by technological progress. Another explanation is that the economy may currently be in the midst of a structural break of the sort Lee et al. [10] found to have occurred in the early and middle parts of the last century. In our model, technological progress evolves smoothly, leading us to believe that the same was true for the time period of our data. It is possible that technology may change abruptly at infrequent intervals, which may be a cause for a structural break.[14] Nevertheless, although the finite nature of our resources may mean that its offsetting effects cannot continue forever, technological progress has been a powerful force in maintaining constant minerals prices over the past three and a half decades.

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Appendix A. Proofs of Propositions 1 and 2

Before proving Proposition 1, we state the following lemma:

Lemma. *With isoelastic decaying costs* (8) *and isoelastic time-varying demand* (11), *if the optimal extraction rate were of the form*:

$$E(t) = gX(t)$$

[14]Another, more speculative, explanation may be that markets in the time period in question acted as if resource extraction could continue forever. Only now, driven by enormous increases in demand (in large parts due to growth in China and India) have markets come to incorporate the binding limits of finite supplies, contributing to recent price spikes.

then the growth rates for different variables would be given by

$$\frac{\dot{X}(t)}{X(t)} = g,$$
$$\frac{\dot{E}(t)}{E(t)} = g,$$
$$\frac{\frac{d\varphi'(E(t))}{dt}}{\varphi'(E(t))} = -\eta g,$$
$$\frac{\frac{d\Psi(X(t))}{dt}}{\Psi(X(t))} = bg,$$
$$\frac{\dot{P}(t)}{P(t)} = -\eta g + a.$$

The results fall simply from direct calculation.

Proof of Proposition 1. From the Maximum Principle, the first-order necessary conditions for a feasible trajectory $\{X^*(t), E^*(t)\}$ to be optimal are

$$[\#1]: \quad p(t) = P(t) - \frac{\partial C(X(t), E(t), t)}{\partial E}, \tag{24}$$

$$[\#2]: \quad \dot{p}(t) = -\frac{\partial C(X(t), E(t), t)}{\partial X} + rp(t), \tag{25}$$

$$[\#3]: \quad \lim_{t\to\infty} p(t)X(t)e^{-rt} = 0. \tag{26}$$

Substituting in Eqs. (10) and (14) into [#1], we get

$$p(t) = P(t) - \frac{\partial C}{\partial E}(X(t), E(t), t) \tag{27}$$
$$= \varphi'(E(t))e^{at} - \Psi(X(t))e^{-\gamma t}. \tag{28}$$

Differentiating with respect to time, we get

$$\dot{p}(t) = \frac{d\varphi'(E(t))}{dt}e^{at} + a\varphi'(E(t))e^{at} - \Psi'(X(t))\dot{X}(t)e^{-\gamma t} + \gamma\Psi(X(t))e^{-\gamma t} \tag{29}$$
$$= \frac{d\varphi'(E(t))}{dt}e^{at} + a\varphi'(E(t))e^{at} - \Psi'(X(t))E(t)e^{-\gamma t} + \gamma\Psi(X(t))e^{-\gamma t}. \tag{30}$$

From [#2],

$$\dot{p}(t) = -\frac{\partial C}{\partial X}(X(t), E(t), t) + rp(t)$$
$$= -\Psi'(X(t))E(t)e^{-\gamma t} + rp(t)$$

which, after rearranging, becomes

$$\dot{p}(t) - rp(t) = -\Psi'(X(t))E(t)e^{-\gamma t}. \tag{31}$$

Substituting (27) and (29) into (31), and simplifying, we get

$$\frac{-\frac{d\varphi'(E(t))}{dt}}{\varphi'(E(t))} + r - a = \frac{(\gamma + r)\Psi(X(t))}{\varphi'(E(t))}e^{-(\gamma+a)t}. \tag{32}$$

By the previously stated lemma, if the solution were of the form $E(t) = gX(t)$, then

$$\frac{\frac{d\varphi'(E(t))}{dt}}{\varphi'(E(t))} = -\eta g.$$

Thus, Eq. (32) reduces to

$$\eta g + r - a = \frac{(\gamma + r)\Psi(X(t))}{\varphi'(E(t))} e^{-(\gamma+a)t}. \tag{33}$$

Since the left-hand side of Eq. (33) is constant, this means that the right-hand side must be constant as well. Thus, it must be the case that the growth rate of the expression on the right-hand side must equal zero:

$$\frac{\frac{d}{dt}\left(\frac{(\gamma + r)\Psi(X(t))}{\varphi'(E(t))} e^{-(\gamma+a)t}\right)}{\frac{(\gamma + r)\Psi(X(t))}{\varphi'(E(t))} e^{-(\gamma+a)t}} = 0$$

which, after simplification, means

$$g(b + \eta) - (\gamma + a) = 0$$

so that

$$g = \frac{\gamma + a}{b + \eta}$$

as desired.

At $t = 0$, Eq. (33) becomes

$$\begin{aligned} \eta g + r - a &= \frac{(\gamma + r)\Psi_0 X_0^b}{A^\eta E_0^{-\eta}} \\ &= \frac{(\gamma + r)\Psi_0 X_0^b}{A^\eta (gX_0)^{-\eta}}, \end{aligned}$$

which, after some rearranging, yields the desired expression (19) for Ψ_0.

In order to satisfy [#3], we need

$$\begin{aligned} 0 &= \lim_{t\to\infty} p(t)X(t)e^{-rt} \\ &= \lim_{t\to\infty} p(t)X_0 e^{(g-r)t} \\ &= \lim_{t\to\infty} \left(P(t) - \frac{\partial C}{\partial E}(X(t), E(t), t)\right) X_0 e^{(g-r)t} \\ &= \lim_{t\to\infty} (P(t) - \Psi(X(t))e^{-\gamma t}) X_0 e^{(g-r)t} \\ &= \lim_{t\to\infty} (P(0)e^{(-\eta g+a)t} - \Psi_0 (X_0 e^{gt})^b e^{-\gamma t}) X_0 e^{(g-r)t} \\ &= \lim_{t\to\infty} (A^\eta E(0)^{-\eta} e^{(-\eta g+a)t} - \Psi_0 X_0^b e^{(gb-\gamma)t}) X_0 e^{(g-r)t} \\ &= \lim_{t\to\infty} (A^\eta (gX_0)^{-\eta} e^{(-\eta g+a)t} - \Psi_0 X_0^b e^{(gb-\gamma)t}) X_0 e^{(g-r)t}. \quad \square \end{aligned}$$

Proof of Proposition 2. From the lemma at the begin of this section, we know that $\frac{\dot{P}(t)}{P(t)} = -\eta g + a$. The result falls from setting this growth rate to zero and substituting Eq. (17) for g. $\square$

Appendix B. Data sources

We test the model using extraction, price, and cost data for 14 subsoil assets: oil, natural gas, brown and hard coal, as well as bauxite, copper, gold, iron, lead, nickel, phosphate, silver, tin, and zinc. The data cover 35 years from 1970 to 2004 and were provided to us by Kirk Hamilton, who compiled them from previously unpublished World Bank sources. We use average annual world prices as calculated in the World Bank data set. These price data correspond to data from Berck and Roberts [2], who provide data from 1870 through 1991.[15] From 1970 to 1991, where the data by the World Bank and by Berck and Roberts overlap, the series are virtually identical. See http://gwagner.com/research/hotelling/ for data and Stata code.

The World Bank data also include average ''rent'' figures, which were calculated as extraction times the difference between price and average cost; we use this formula to calculate average costs. Assuming an isoelastic form of the cost function, as in Eq. (8), average and marginal cost are the same. Extraction data by commodities are available for between 38 and 94 countries.

The data set also includes reserve data. We deem these as the least reliable. For one, reserves for some minerals and years are defined as ''reserves''; others are defined as ''reserve base.'' While exact definitions are not available, reserve bases are generally larger and less precisely estimated than reserves. More importantly, though, reserve and reserve base refer to minerals still left in the ground. Stock $X(t)$ in our model, defined by Eq. (1), refers to the cumulative stock already extracted. There is no apparent relationship between the two quantities, which would enable us to calculate $X(t)$ based on reserve or reserve base estimates given in the data. Instead, we estimate $X(t)$ by setting $X(0) = 0$ in Eq. (1) and using extraction $E(t)$ for the years 1970–2000. This normalization biases $X(t)$ downward and possibly overplays its effect on marginal extraction costs in Eq. (8) in early years while underestimating it in later years.

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